NASA CR-1299

# NASA CONTRACTOR REPORT

NASA CR-1299



# SONIC BOOM PROPAGATION IN A STRATIFIED ATMOSPHERE, WITH COMPUTER PROGRAM

*by Wallace D. Hayes, Rudolph C. Haefeli, and H. E. Kulsrud*

*Prepared by*
AERONAUTICAL RESEARCH ASSOCIATES OF PRINCETON, INC.
Princeton, N. J.
*for Langley Research Center*

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • APRIL 1969

NASA CR-1299

# SONIC BOOM PROPAGATION IN A STRATIFIED ATMOSPHERE, WITH COMPUTER PROGRAM

By Wallace D. Hayes, Rudolph C. Haefeli, and
H. E. Kulsrud



Prepared under Contract No. NAS 1-7721 by
AERONAUTICAL RESEARCH ASSOCIATES OF PRINCETON, INC.
Princeton, N.J.

for Langley Research Center

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 – CFSTI price $3.00

CONTENTS

# SONIC BOOM PROPAGATION IN A STRATIFIED ATMOSPHERE, WITH COMPUTER PROGRAM

Wallace D. Hayes*, Rudolph C. Haefeli, and H. E. Kulsrud
Aeronautical Research Associates of Princeton, Inc.

## SUMMARY

An analysis is presented of the propagation of sonic boom in a horizontally stratified atmosphere with winds. This analysis, to some extent a synthesis of established theory but with many new features, is given in sufficient detail to serve as an algorithm for the computation of sonic boom pressure signatures. This algorithm is realized in a FORTRAN computer program.

Required inputs include atmospheric properties and horizontal winds as functions of altitude, information on the flight path of the maneuvering aircraft, and aircraft F-functions. Ray-tube areas are computed according to geometric acoustic theory, and nonlinear effects are accounted for through an appropriate age variable. The output includes midfield pressure signatures at any altitude.

Results from sample calculations are presented and discussed.

## INTRODUCTION

Sonic booms have become of prime importance in the design and operation of supersonic aircraft. A need has been felt for a comprehensive analysis and algorithm, realized in a practicable computer program, which would provide realistic calculations for sonic boom signatures in our atmosphere. The project reported here was to carry out such an analysis with computer program.

Earlier algorithms for sonic boom have used various unjustified simplifying assumptions. A basic aim of the present algorithm has been to avoid these assumptions as far as possible and to extend the cases which could be considered. Thus, the present algorithm includes the following features:

(1) The inclusion of maneuvering aircraft in a sonic boom pressure calculation;

(2) An appropriate ray-tube area calculation based on linear geometric acoustics;

*Professor of Aerospace Sciences, Princeton University

(3) Results in the form of complete signatures, without far-field assumptions, obtained through the use of an "age" variable in the calculation of nonlinear effects.

The present algorithm assumes a horizontally stratified atmosphere with horizontal winds but without turbulence. This limiting assumption corresponds to the case of greatest practical interest and considerably simplifies the calculation.

The analysis is largely a rational synthesis of existing theories described in the literature, with some new theoretical development. Specific references are cited in the body of the report. A principal new theoretical development is in the calculation of ray-tube area. The analysis is also new in the careful piecing together of a number of calculations, principally in the relation of the wave system and rays issuing from the aircraft with a wave system and rays properly describing propagation in the stratified atmosphere with winds. This relation requires the consideration of a galilean transformation connecting a local coordinate system with the fixed coordinate system.

A note of caution at this point may be in order. Although our analysis may be described as largely a synthesis of existing theories, it should be pointed out that not all these theories may be familiar to all workers in the field of sonic boom. In order to make the analysis feasible, the concept of galilean invariance has been brought in from the subject of mechanics, and a number of concepts have been brought in from the general theory of wave propagation. The pertinent literature is diffused through many sources. A number of basic papers were written in contexts different from that of sonic boom. Thus, some readers will not find our analysis as a synthesis of the theories with which they are well acquainted. In general, the more familiar sonic boom theories are inadequate.

The digital computer program has been written in ASA FORTRAN IV (except some literal text enclosed in asterisks) with flexibility a main aim. The program is designed to be usable on a wide variety of modern computers and to be applicable to a variety of problems. It was developed using an IBM-1130, Model 2B, and then modified for and operated with a CDC-6600. The program may be altered to accommodate the operating system constraints of a particular computer through simple changes in input-output unit designation. It may also be necessary to make some alterations in program structure from subprogram linkage to main program linkage to meet core storage requirements, as in the case for the IBM-1130. A number of input options have been provided. There are choices in the specifications of input and output units, in how the atmosphere is to be specified, and in how certain maneuver time derivatives are to be obtained from input data.

This report is accordingly divided into two main parts - one giving an exposition of the basic theory and development of the equations, the other describing and listing the computer program and presenting sample results. The first part occupies the chapter entitled THEORETICAL ANALYSIS. This begins with a general description of the theory, with accent on the physical reasoning and motivation underlying the analysis. In the course of the analysis, brief statements are included on its applications in the computer program. Besides current references, there are some historical notes appended.

The second part, consisting of COMPUTER PROGRAM and COMPUTATION RESULTS, includes a complete description of the program, with tables giving the FORTRAN nomenclature used for various variables and subroutines, and with a program listing. Sample input and output listings are included, and typical computation results are presented.

## SYMBOLS

This section includes symbols used in the analysis, excepting a few which are only used where they are defined. FORTRAN symbols that are employed in the computer program are identified in Table 1.

| Symbol | | Page No. |
|---|---|---|
| $a$ | speed of sound (eq. 6) | 21 |
| $A$ | ray-tube area cut by horizontal plane (eqs. 19, 26) | 34 |
| $A(x_o)$ | area distribution of slender body | 43 |
| $\bar{c}$ | group or ray velocity (eq. 12) | 29 |
| $c_n$ | normal phase velocity (eq. 11) | 28 |
| $c_o$ | Snell's law invariant (eqs. 14, 16) | 29 |
| $c_p$ | specific heat at constant pressure | 65 |
| $C_D$ | wave drag coefficient (eq. 36) | 49 |
| $C_{D\phi_r}$ | drag coefficient per unit azimuth angle (eq.36) | 49 |
| $C_L$ | lift coefficient (eq. 4) | 18 |

| | | |
|---|---|---|
| $C_T - C_D$ | net axial force coefficient (eq. 3) | 18 |
| D | drag (eq. 3) | 18 |
| $f_y, f_z$ | line force distributions | 43 |
| F | F-function for aircraft signatures (eq. 33) | 46 |
| $F_i$ | input F-function | 46 |
| $F_f$ | F-function conversion factor (eq. 34) | 48 |
| g | gravitational acceleration (eq. 1) | 16 |
| $H_G$ | altitude of ground above sea level | 16 |
| $I_{1,2,3}$ | integrals used in calculation of A (eq. 24) | 39 |
| k | heat conduction coefficient (eq. 58) | 63 |
| $K_R$ | reflection factor | 64 |
| $\ell, m, n$ | direction cosines of initial wave normal (eq.8) | 25 |
| $\ell(x_1)$ | equivalent line force distribution | 45 |
| L | lift (eq. 4) | 18 |
| L | distance along aircraft axis (local phase) | 48 |
| $L_a$ | aircraft reference length | 48 |
| M | Mach number of aircraft, V/a | 21 |
| $\bar{n}$ | unit vector normal to wave front | 28 |
| $n_L, n_T$ | normal and axial load factors (eqs. 3, 4) | 18 |
| $N(\tau)$ | reduced longitudinal kinematic viscosity (eq. 60) | 66 |
| p | pressure (eqs. 1, 39) | 16 |
| q | perturbation velocity (eq. 39) | 51 |
| $q_\infty$ | dynamic pressure (eq. 3) | 18 |
| r | cylindrical radius in local coordinates | 44 |
| $\bar{r}$ | position vector (eq. 12) | 29 |

| | | |
|---|---|---|
| $\bar{r}'$ | horizontal position vector | 36 |
| R | gas constant; hyperbolic radius | 44 |
| s | distance normal to wave front (local phase) | 56 |
| $S_{ref}$ | reference wing area for force coefficients (eqs. 3, 4) | 18 |
| $S(x_1,\phi_r)$ | area distribution of equivalent body of revolution | 45 |
| $S_o(\xi)$ | integral of $V_E(\xi)$ (eq. 54) | 63 |
| $\mathcal{S}(\xi_1)$ | integral of $V_E(\xi_1,\tau)$ (eq. 51) | 58 |
| $t(x_o,y_o)$ | wing thickness | 44 |
| t | time along ray (eq. 18) | 26 |
| $t_a$ | time along aircraft trajectory (eq. 2) | 18 |
| T | absolute temperature | 16 |
| T | thrust (eq. 3) | 18 |
| $\bar{u}$ | wind velocity $(-u_x,-u_y,0)$ (eq. 2) | 16 |
| $u_n,u_t$ | minus components of $\bar{u}$ in $(x_1,y_1)$ coordinates (eq. 15) | 31 |
| V | aircraft velocity relative to atmosphere, Ma (eq. 2) | 17 |
| $V_E$ | measure of signal invariant on kinematic ray (eqs. 40, 45) | 52 |
| W | weight of aircraft (eqs. 3, 4) | 18 |
| x,y,z | fixed coordinate system; east, north, and above ground, respectively | 16 |
| $x',y',-z$ | coordinate system aligned with aircraft velocity | 22 |
| $x_1,y_1,-z$ | coordinate system aligned with wave normal (eq. 19) | 31 |

| | | |
|---|---|---|
| $x, y, z$ | local coordinates near aircraft | 43 |
| $x_o, y_o, z_o$ | dummy coordinates near aircraft | 43 |
| $x_1$ | axial coordinate for equivalent body of revolution | 45 |
| $\beta$ | Prandtl-Glauert parameter, $(M^2 - 1)^{1/2}$ | 43 |
| $\beta$ | inverse of atmospheric scale height | 59 |
| $\gamma$ | aircraft climb angle (eq. 2) | 17 |
| $\gamma_e$ | ratio of specific heats | 16 |
| $\Delta, \delta$ | perturbation from undisturbed value | 39,28 |
| $\eta$ | wind heading angle (whence wind blows) | 16 |
| $\theta$ | inclination angle of $\bar{n}$ below horizontal (eqs. 10, 17) | 25 |
| $\mu$ | Mach angle, $\sin^{-1}(1/M) = \tan^{-1}(1/\beta)$ | 22 |
| $\mu, \mu'$ | shear and dilatational viscosities (eq. 58) | 65 |
| $\nu$ | heading angle of wave normal $\bar{n}$ (eq. 9) | 25 |
| $\xi$ | linear phase variable (time) (eq. 41) | 51 |
| $\xi_1$ | actual phase variable (time) (eq. 49) | 56 |
| $\rho$ | atmospheric density (eq. 1) | 16 |
| $\tau$ | age (eq. 46) | 57 |
| $\phi$ | azimuth angle of wave normal from vertical plane | 22 |
| $\phi_a$ | aircraft bank angle | 18 |
| $\phi_r$ | azimuth angle of wave normal relative to aircraft | 20 |
| $\Phi$ | local perturbation velocity potential (eq.30) | 43 |
| $\psi$ | heading angle of aircraft (eq. 2) | 17 |

Subscripts

| | | |
|---|---|---|
| a | aircraft | 17 |
| o | initial value at time of emission of a ray from aircraft | 32 |
| w | wing | 45 |

Vector Components

| Vector | | Coordinate Systems | | |
|---|---|---|---|---|
| Name | Symbol | $(x,y,z)$ | $(x',y',-z)$ | $(x_1,y_1,-z)$ |
| Position | $\bar{r}$ | $x,y,z$ | $x',y',-z$ | $x_1,y_1,-z$ |
| Horizontal unit, east | $\bar{i}$ | $1,0,0$ | $\sin\psi,-\cos\psi,0$ | $\sin\nu,-\cos\nu,0$ |
| Horizontal unit, north | $\bar{j}$ | $0,1,0$ | $\cos\psi,\ \sin\psi,0$ | $\cos\nu,\ \sin\nu,0$ |
| Vertical unit | $\bar{k}$ | $0,0,1$ | $0,0,-1$ | $0,0,-1$ |
| Horizontal position | $\bar{r}'$ | $x,y,0$ | $x',y',\ 0$ | $x_1,y_1,0$ |
| Aircraft velocity | $\bar{V}$ | | $V\cos\gamma,0,$ $-V\sin\gamma$ | |
| Initial wave normal | $\bar{n}_o$ | | $\ell,m,n$ | $\cos\theta_o,0,$ $\sin\theta_o$ |
| Wave normal | $\bar{n}$ | | | $\cos\theta,0,\sin\theta$ |
| Wind | $\bar{u}$ | $-u_x,-u_y,0$ | | $-u_n,-u_t,0$ |
| Horizontal unit, propagation | $\bar{n}'$ | $\sin\psi,\cos\psi,0$ | | $1,0,0$ |
| Inverse phase velocity | $c_n^{-1}\bar{n}$ | | | $c_o^{-1},0,c_o^{-1}\tan\theta$ |
| Ray or group velocity | $\bar{c}$ | | | $a\cos\theta-u_n,$ $-u_t,\ a\sin\theta$ |

PRECEDING PAGE BLANK NOT FILMED.

# THEORETICAL ANALYSIS

## General Description

The intent of this section is to present a guide to the theoretical analysis which will be developed in this chapter. This guide is presented in several subsections. The first gives a brief description of the nature of sonic boom theory. The next three discuss certain basic concepts of geometric acoustics, with one purpose being that of explaining the basis for the assumption of steady ray geometry on which the entire analysis is based. The last describes the detailed analysis in digest form, essentially section by section.

The nature of sonic boom theory.- Sonic boom is an acoustic phenomenon. The appropriate theory for sonic boom propagation is an acoustic theory with both simplifications and complications which do not normally appear in acoustic theory. With certain exceptions, the appropriate theory is the theory of geometric acoustics, analogous to geometric optics. The theory of geometric acoustics is valid in an asymptotic sense when the wave length is small compared with characteristic macroscopic scales of the problem. Such macroscopic scales include the radii of curvature of the wave fronts and the scale height (e.g., $p/\rho g$) of the atmosphere. (Symbols are defined in the list of SYMBOLS.) Geometric acoustics is invalid in the region near the aircraft, where a separate treatment is needed to obtain initial conditions for the propagated signal.

Standard acoustic theories are linear. In sonic boom propagation, nonlinear effects are locally very weak, but they have a nonnegligible cumulative effect during propagation over large distances. The cumulative nonlinear effect comprises distortion of the signal and the production of shock waves. We can thus describe sonic boom theory as an application of geometric acoustics, with a particular matching theory for initial conditions and with a modification for nonlinear effects. A recent review of the theoretical approach to sonic boom which is here developed in detail may be found in reference 1.

In any sonic boom theory with the generality of the theory presented here, the concepts of galilean transformations and of phase necessarily appear. These concepts are discussed below.

Coordinate systems and galilean transformations.- Two principal coordinate systems are required in our theory. One is an unaccelerated coordinate system fixed relative to the ground, and the propagation through the atmosphere is treated in this coordinate system. The other, defined locally at a particular instant, is an unaccelerated coordinate system aligned with the aircraft

flight axis and moving with the aircraft velocity at the instant of interest. The flow near the aircraft is conveniently described in this coordinate system. These two coordinate systems are related through a galilean transformation. A galilean transformation is a transformation from one unaccelerated coordinate system to another moving relative to the first at a constant velocity. A quantity is galilean invariant if it does not change under a galilean transformation.

In one particular step of the analysis the consideration of the galilean transformation is inescapable. This step appears when the variables describing the (local) flow near the aircraft are transformed into the appropriate variables describing the (global) acoustic propagation in the coordinate system fixed relative to the ground. In this critical step we shall avoid going through the formal details of the galilean transformation. Instead we identify corresponding variables which are inherently galilean invariant; by relating these to both the local and global variables of the problem, we are able to connect the local with the global variables. This stratagem simplifies this critical step considerably and, in effect, accomplishes the inescapable galilean transformation in a relatively easy way. No other feasible way of relating the local and global variables was discovered.

In this report we are concerned primarily with the case of a horizontally stratified atmosphere with winds. Such an atmosphere remains horizontally stratified under a horizontal galilean transformation, one in which the relative velocity is horizontal. Hence, any theory for this case must be invariant under such a transformation. This property has been used in the development of the analysis presented here to check it for algebraic consistency.

In general, consideration of which variables are galilean invariant was of great help in the development of the analysis presented in this report. A quantity which is galilean invariant is independent of the choice of coordinate system. It is found that the analysis is simpler, both algebraically and conceptually, when such variables are chosen to describe the solution. Thus the consideration of galilean invariance has guided the general course of the analysis and the specific choice of variables used.

In this report, we mention in a number of places whether particular variables are or are not galilean invariant. Except in the critical step mentioned above, the reader uninterested in this property may ignore the mention. In the critical step where the galilean transformation is inescapable, the galilean invariance of the pertinent variables is essential to the analysis. This step appears in the section entitled Geometric Acoustics and Blokhintsev Invariance.

Wave fronts and phase.- According to the basic concepts of acoustics, the signal is propagated on wave fronts. Wave fronts are surfaces that move through space and are characteristic surfaces for the complete hydrodynamic equations (more precisely, they are characteristic hypersurfaces in space-time). A wave system includes a one-parameter family of wave fronts. A variable that parametrizes the wave fronts is termed a phase. Accordingly, the phase is the principal independent variable in terms of which an acoustic signal or pressure signature is described. Any monotonic differentiable function of a phase variable is also a phase variable, as it will serve equally well to parametrize the wave fronts. Since the only purpose of the phase is to label wave fronts, what its dimensions may be is unimportant. A phase may be chosen to be dimensionless or to have the dimensions of time or distance, as may be convenient.

As defined, the phase parametrizes the wave fronts over the entire history of the wave propagation and is, in this sense, a global variable. The word "phase" used alone refers to this global concept, although to emphasize this property we occasionally use the term global phase. It is convenient to distinguish from this concept the concept of a local phase, defined to be any variable in terms of which an acoustic signal may be expressed locally. A local phase is not generally a phase in the global sense. A microphone fixed in space records pressure as a function of time as a wave system goes by. Thus, time measured from the passage of a reference wave front is a phase variable, one that turns out to be global as well as local in a steady atmosphere; this particular variable is the one we shall use in our general treatment of geometric acoustics. Distance measured at a given instant normal to the wave fronts from a reference wave front is a suitable local phase. Distance aft of a reference Mach cone in a coordinate system fixed with respect to the aircraft is another local phase, and is the one we shall use in treating the flow near the aircraft.

To illustrate the distinction between phase and a local phase, we consider the particular variable distance normal to wave fronts. The distance between two wave fronts in an atmosphere either with winds or with nonuniform speed of sound does not remain constant as the fronts move. A wave front 10 feet from the reference front at one time will be different from the front found 10 feet from the reference front at some different time. Thus, distance from the reference front is not a global phase, even though it can be used as a local phase.

The reason the concept of phase is important is that we must correctly identify the wave fronts over the entire history of the wave propagation. A phase variable, correctly defined globally, serves precisely this purpose. In our presentation of the theoretical analysis, we use two local phases $(L/L_a$ and $s)$ as well as

a basic global one ($\xi$). We may note that in the general case in which the atmospheric properties change with time, it is impossible to use a physically defined entity (e.g., time measured by a fixed observer) as the (global) phase. In this case the phase must be defined as a variable in its own right, with no generally valid physical interpretation.

We pick a particular reference wave front as the front of zero phase. This wave front is a surface in space which is tangent at the aircraft to the Mach cone with vertex at some specified reference point on the aircraft axis. Such a surface is termed a Mach conoid. For convenience, we consider the reference point to be at the nose of the aircraft. Other points on the aircraft axis are at the vertices of Mach conoids or wave fronts of different phase. These concepts are discussed further in the section on Mach Conoids and Ground Intersections.

Geometric acoustics and rays.- The basic concept of geometric acoustics is that the signal is propagated along rays. Rays are trajectories of points moving in space. Each ray moves with a wave front, and the concept of the propagation of a signal on rays is consistent with that of its propagation on wave fronts. Since a ray is a point trajectory, that is, a specification of the motion of a point with time, it is a kinematic rather than a geometric entity. Where it appears desirable to emphasize this character, we term a ray a kinematic ray. The path of a ray is a geometric entity. When a number of rays traverse the same path, we term the path a geometric ray. Since phase is constant on each wave front and each ray moves with a wave front, phase is also constant on rays. In a general solution the rays form a three-parameter family of point trajectories. The three parameters are analogous to Lagrangian coordinates for particles moving in a fluid flow. One of the parameters is the phase, while the other two are selected to be an azimuth angle $\phi$ and a time $t_a$ (to be defined later).

In general, the rays corresponding to values of the phase other than zero do not follow the same paths through space as do the rays for which the phase is zero. An important special case is that in which the ray geometry is steady, in which every ray path is the path for a one-parameter family of kinematic rays. In this case the ray paths are what we have termed geometric rays which form a two-parameter family of curves in space. In applying the analogy to particles moving in a fluid flow to this case, the flow is assumed steady, with the geometric rays then analogous to streamlines. The property of steady ray geometry is not galilean invariant, and this fact indicates that the assumption of this property must be made with care.

Historically, for the most part only this special case has been considered. Moving sources are rarely considered in geometric

optics; moving acoustic sources are generally treated in a coordinate frame in which they are fixed and, generally, only aircraft in steady flight have been considered as generators of sonic boom. Thus, historically, rays have been considered primarily as geometric entities.

We make the assumption of steady ray geometry in the sonic boom problem, with $t_a$ and $\phi$ as the parameters for the geometric rays. This assumption is justified by the thinness of the entire wave system of interest, essentially by the fact that the aircraft length is small compared with other macroscopic characteristic scales. A ray emanating from the tail is simply so close to the corresponding one of zero phase that the difference in their ray paths may be neglected. If $L_a$ is a measure of the thickness of the wave system and $R$ a macroscopic scale measure, the required condition is $L_a/R \ll 1$ . If $\lambda$ is a measure of the characteristic wave length of the acoustic signal, it is the condition $\lambda/R \ll 1$ which justifies geometric acoustics. The sonic boom problem is unique among acoustic problems in having $\lambda \approx L_a$, with the consequence that the steady-ray-geometry assumption is valid when geometric acoustics is valid. Thus this assumption is sound even though the problem with a maneuvering aircraft is not a steady one.

This assumption is basic to our analysis. It permits our calculating only the two-parameter family of rays corresponding to zero phase, considering the aircraft to be a single moving point in space. Another basic assumption is that the cumulative nonlinear effects do not affect the ray geometry. This is discussed later when we treat the nonlinear distortion. Hence, ray calculations follow linear theory.

Besides the concepts of wave fronts, phase, and rays, another basic concept in geometric acoustics is that of ray tubes and ray-tube areas. Although ray tubes may be defined in the general case, they are much easier to visualize with the steady-ray-geometry assumption. In the neighborhood of a given geometric ray, we visualize a tube of geometric rays, i.e., a ray tube. The corresponding entity in the analogous steady fluid flow is a streamtube. A ray-tube area is a measure of the differential area intercepted by a surface cutting the ray tube and may be considered a vector quantity. Like a streamtube, a ray tube is a differential quantity, and a ray-tube area is actually defined in terms of derivatives with respect to the ray parameters.

An element that greatly simplifies the calculation is the assumption that the atmosphere with its winds is horizontally stratified (layered). A refraction law of the type of Snell's law in geometric optics then holds. This law permits the calculation of both the rays and corresponding ray-tube areas by quadratures.

Digest of the theoretical analysis.- We turn now to a general description of our analysis of sonic boom, the details of which are presented in the subsequent sections of this chapter. The analysis may be conceptually divided into three main parts which will appear in sections of the chapter preceded by a short section on The Atmosphere; at the end we add a Note on Viscous Effects. The first part of the analysis comprises the sections entitled Aircraft Maneuvers, Initial Wave Normals, Mach Conoids and Ground Intersections, Snell's Law and Ray Tracing, and Ray-Tube Area. It concerns the calculation of the rays and ray-tube areas for zero phase (the reference phase). The second part comprises the sections entitled Flow Near the Aircraft, and Geometric Acoustics and Blokhintsev Invariance. It concerns the calculation by linear theory of acoustic signals along each geometric ray. The third part comprises the sections entitled Signal Distortion and Age Variable, and Shock Location. It concerns the calculation, with shocks properly accounted for, of the nonlinear distortion of the signal. A number of vector quantities are introduced and used in the analysis. The components of these vectors in the various coordinate systems used are given in the section entitled SYMBOLS, Vector Components.

The maneuver of the aircraft (strictly speaking, of the reference point) is required in detail. Variables are introduced in the section on Aircraft Maneuvers which describe the trajectory in space, the orientation of the flight axis, the velocity of the aircraft relative to the local atmosphere, and the local sound speed, all as functions of time $t_a$. Time derivatives of certain of the variables are also determined, for later use in the ray-tube area calculation. At each instant $t_a$ we visualize a Mach cone attached to the nose of the aircraft. The normals to the Mach cone form a one-parameter family of directions forming a wave-normal cone with the parameter being an azimuth angle $\phi$. The two quantities $t_a$ and $\phi$ are the ray parameters discussed earlier.

In the section Mach Conoids and Ground Intersections, the wave fronts and rays from an aircraft in maneuvering flight are discussed generally, with particular attention to the intersections of the rays and wave fronts with the ground.

The generators of the wave-normal cone at the aircraft are the initial wave normals for the calculation of the rays. The orientation of these normals is known as a function of the ray parameters. For each wave normal we calculate two quantities which are invariant on rays according to the appropriate Snell's law. These invariants are then used to calculate the ray trajectories.

The ray-tube area is defined as that given by horizontal cutting planes. An analytic expression for this area is obtained

in terms of the maneuver variables, certain of their time derivatives, and three quadratures along the ray. The ray-tube area is thus obtained as a function of altitude along each ray and may be calculated concurrently with the ray trajectory.

For the second part of the analysis (Flow Near the Aircraft, Geometric Acoustics and Blokhintsev Invariance), we consider first the flow close to the aircraft. In particular, we need the asymptotic form of the local solution, valid at a distance from the flight axis large compared with the effective lateral dimensions of the aircraft but small compared with characteristic scales for the atmosphere. This asymptotic form of the local solution is interpretable as a geometric acoustics solution. At a sufficiently large distance $r$ in a particular direction away from the flight axis of the aircraft, the solution appears the same as that from a line distribution of sources and sinks, the same as that from an equivalent body of revolution. The pressure perturbation in the asymptotic solution is proportional to $r^{-1/2}$ times a function $F$ of a suitable defined phase and of an azimuth angle $\phi_r$ (simply related to $\phi$). This F-function depends also upon the Mach number and lift coefficient of the aircraft, which are functions of the time $t_a$. The F-function is then a function of phase and of the ray parameters $t_a$ and $\phi$ and is invariant along each kinematic ray. It is obtainable either by a computation (outlined here in the section Flow Near the Aircraft) or from experiment. It is assumed to be a known function in the computer program.

In the general stratified atmosphere with winds, the appropriate general definition of phase is as the time $\xi$ measured by an observer fixed in a ground-based coordinate system and defined to be zero the instant the zero-phase wave front passes. Invariance results of Blokhintsev permit the acoustic signal of each ray to be described in terms of a function $V_e(\xi)$ which is constant on the ray.

The relation between the local and general phase variables is then found so that $F$ may be expressed as a function of $\xi$. The relation between the function $F$ and $V_e$ is also found, which then gives the function $V_e(\xi)$ for each ray (since $F$ is assumed known). The relation between $V_e$ and pressure perturbation $\Delta p$ is known, so that $\Delta p(\xi)$ is determined at any point on each geometric ray.

In the third part of the analysis (Signal Distortion and Age Variable, Shock Location), we consider the change in propagation speed proportional to the strength of the signal. This nonlinear effect does not, in principle, influence the magnitude of the pressure perturbation in the acoustic signal. Rather it causes phase shifts in the signal, whereby a given point in the signature may appear earlier or later than predicted by the linear theory.

This phase distortion arises because compression waves travel slightly faster, and expansion waves slightly slower, than do infinitesimal disturbances. In terms of the phase variable $\xi$, this phase shift equals $V_e$ times an age variable $\tau$ which can be computed along each ray by a quadrature. The distorted signal appears as the original one $V_e(\xi)$ sheared by an amount proportional to $\tau$.

The distorted signal may be multivalued and may thus give several values of the pressure perturbation for a single value of $\xi$. Physically, this anomaly indicates the presence of shock waves and disappears when shocks are properly taken into account. A separate analysis shows where shock waves must lie and shows which parts of the signature have been "eaten up" by the shocks and no longer appear. The result of the analysis is the complete, single-valued pressure signature at any desired point, with shocks shown if they are present. The nonlinear effect does affect the magnitude of the pressure perturbation insofar as the parts of the original signal that are eaten up by shocks no longer appear.

The theory fails near a caustic, a surface in space at which the ray-tube area becomes zero. It also fails near the boundary of a shadow zone into which no rays penetrate and may fail near a critical ray for which the F-function is singular in some way. The linear solutions in these regions are solutions to diffraction problems for which geometric acoustics is invalid. These problems are outside the scope of the analysis of this report.

## The Atmosphere

The coordinate system used is cartesian with $x$ and $y$ horizontal distances east and north, respectively, from a reference origin on the ground. The ground is assumed level at an altitude $H_G$ above sea level, and $z$ is altitude above the ground. The atmosphere is assumed to be a calorically perfect gas (constant specific heat ratio $\gamma_e$) with the thermodynamic properties temperature $T$, density $\rho$, pressure $p = RT\rho$, and speed of sound $a = (\gamma_e RT)^{1/2}$ given as functions of altitude. The pressure obeys the hydrostatic law

$$\frac{dp}{dz} = -\rho g \tag{1}$$

with $g$ the acceleration due to gravity. Winds are horizontal with magnitude $u$ and direction dependent only upon $z$. The wind direction is specified by the wind heading angle $\eta$ measured clockwise from north. In accord with ancient historical convention, the wind heading is taken as the direction from which the

wind comes ("the north wind doth blow"), and we have acceded to this convention. The velocity of the wind has east and north components $(-u_x, -u_y)$ with

$$u_x = u \sin \eta$$

$$u_y = u \cos \eta$$

Application in the program.- Inputs into the program include the temperature $T$, either the density $\rho$ or the pressure $p$, the wind speed $u$, and the wind heading angle $\eta$ given as functions of the altitude $z + H_G$ above sea level. Also input is the ground altitude $H_G$. An option provides for specification of the 1962 U.S. Standard Atmosphere (ref. 2) with any wind distribution.

There is no provision in the program to ensure that the hydrostatic relation (1) is satisfied. Hydrostatic consistency of the input data is the responsibility of the operator.

## Aircraft Maneuvers

In our study of sonic boom, we need the trajectory of the aircraft in order to know where the rays start. The equations to be integrated for the aircraft trajectory, if it is not specified, are presented in this section. The time derivatives of heading angle, climb angle, and aircraft velocity or Mach number are needed later in the calculation of ray-tube area. Equations permitting the calculation of these derivatives from the aircraft load factors are also included here.

The aircraft moves through space supersonically on some known trajectory. This trajectory is described by the coordinates $x_a(t_a)$, $y_a(t_a)$, and $z_a(t_a)$ of a reference point on the aircraft, which we choose to be the aircraft nose. The subscript $a$ identifies variables defining the aircraft position in a ground-fixed coordinate system.

The first stage of our analysis is an investigation of the equations governing the trajectory of the aircraft. The aircraft has velocity $\overline{V}$ measured in a coordinate system in which the local atmosphere is at rest (i.e., in a coordinate system moving with the wind velocity). This velocity has magnitude $V$, a heading angle $\psi$ measured clockwise from north, and a climb angle $\gamma$ above the horizontal. The direction of $\overline{V}$ at any instant is termed the flight axis.

With respect to the ground-fixed coordinate system, the differential equations for the flight trajectory are then

$$\frac{dx_a}{dt_a} = V \cos \gamma \sin \psi - u_x(z_a)$$

$$\frac{dy_a}{dt_a} = V \cos \gamma \cos \psi - u_y(z_a) \qquad (2)$$

$$\frac{dz_a}{dt_a} = V \sin \gamma$$

If $V$, $\gamma$, and $\psi$ are known functions of $t_a$, we can integrate the third equation to obtain $z_a(t_a)$, and then obtain $x_a$ and $y_a$ by integrating the other two.

The acceleration of the aircraft is equal to the net force divided by the aircraft mass $W/g$. The component along the flight path is $(n_T - \sin \gamma)g$, where $n_T$ is a net thrust load factor defined by

$$n_T = (T - D)/W = (C_T - C_D)q_\infty S_{ref}/W \qquad (3)$$

with $T$ and $D$ representing the thrust and drag on the aircraft, respectively, $C_T$ and $C_D$ the thrust and drag coefficients, $q_\infty$ the dynamic pressure $(1/2)\rho V^2$, and $S_{ref}$ the aerodynamic reference area. The quantity $g \cos \gamma$ is a component of the acceleration due to gravity acting laterally, or normal to the flight direction.

The aircraft is assumed to be laterally symmetric, without side forces, and to be banked at an angle $\phi_a$ about the flight axis. The lift on the aircraft then provides a normal acceleration component $n_L g$, where $n_L$ is the lift load factor defined by

$$n_L = L/W = C_L q_\infty S_{ref}/W \qquad (4)$$

where $L$ and $C_L$ represent the lift and lift coefficient, respectively. This is directed so that $n_L g \cos \phi_a$ opposes the gravity component $g \cos \gamma$, while $n_L g \sin \phi_a$ is horizontal. Figure 1 shows the acceleration components while figure 2 shows how $\phi_a$ is defined and shows the lateral acceleration components. The coordinate frame $(x',y',z)$ is rotated $\pi/2 - \psi$ counterclockwise relative to the reference frame $(x,y,z)$ and is used later to develop wave propagation directions.

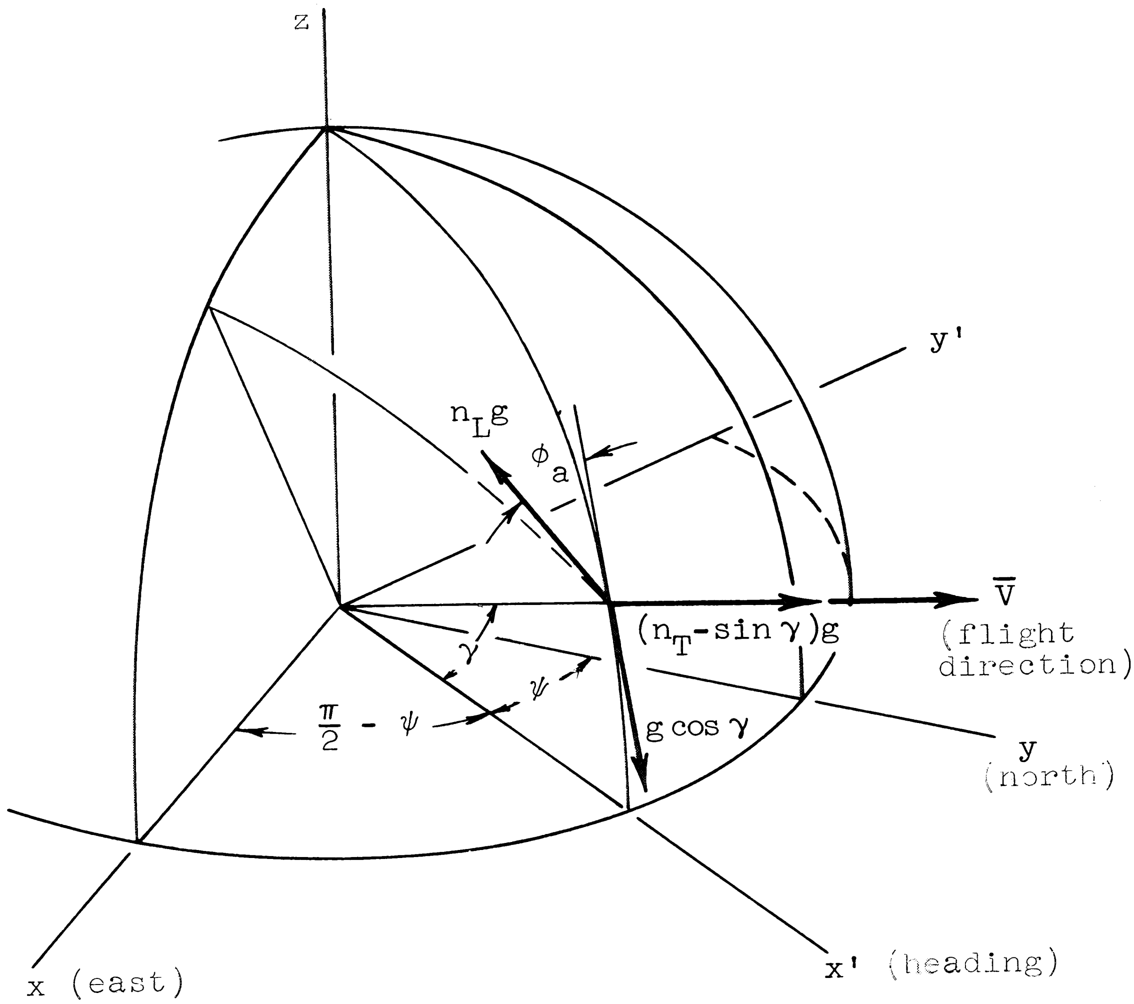



Figure 1. Acceleration diagram for maneuvering aircraft.

To obtain expressions for the lateral and axial acceleration components, we differentiate equations (2) with respect to $t_a$ and combine terms. This yields

$$n_L g \sin \phi_a = \cos \psi \frac{d^2 x_a}{dt_a^2} - \sin \psi \frac{d^2 y_a}{dt_a^2}$$

$$(n_L \cos \phi_a - \cos \gamma)g = \cos \gamma \frac{d^2 z_a}{dt_a^2}$$

$$- \sin \gamma \left( \sin \psi \frac{d^2 x_a}{dt_a^2} + \cos \psi \frac{d^2 y_a}{dt_a^2} \right)$$

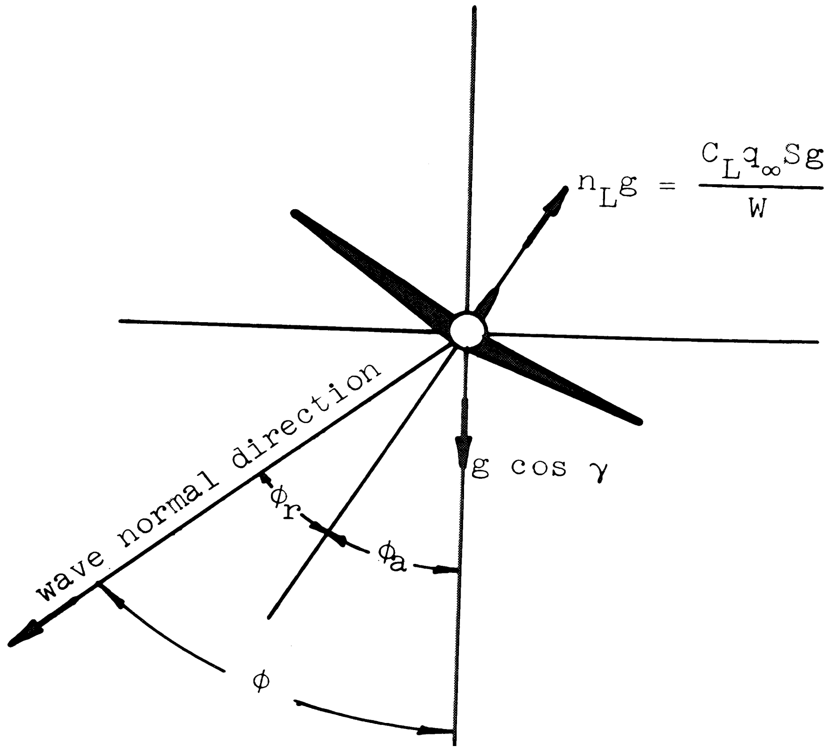



Figure 2. View looking forward along flight axis showing acceleration components, bank angle, and azimuth angle.

$$(n_T - \sin\gamma)g = \sin\gamma \frac{d^2 z_a}{dt_a^2}$$

$$+ \cos\gamma\left(\sin\psi \frac{d^2 x_a}{dt_a^2} + \cos\psi \frac{d^2 y_a}{dt_a^2}\right)$$

Carrying out the differentiation with respect to $t_a$ , we then obtain

$$V \cos \gamma \frac{d\psi}{dt_a} = n_L g \sin \phi_a - \frac{dz_a}{dt_a}\left(\sin \psi \frac{du_y}{dz} - \cos \psi \frac{du_x}{dz}\right)$$

$$V \frac{d\gamma}{dt_a} = (n_L \cos \phi_a - \cos \gamma)g$$

$$- \frac{dz_a}{dt_a} \sin \gamma\left(\cos \psi \frac{du_y}{dz} + \sin \psi \frac{du_x}{dz}\right) \quad (5)$$

$$\frac{dV}{dt_a} = (n_T - \sin \gamma)g + \frac{dz_a}{dt_a} \cos \gamma\left(\cos \psi \frac{du_y}{dz} + \sin \psi \frac{du_x}{dz}\right)$$

The factor $dz_a/dt_a$ is, from (2), simply $V \sin \gamma$ . Equations (5) relate the two load factors $n_L$ and $n_T$ with the time derivatives of $\psi$ , $\gamma$ , and $V$ .

Although they are not directly involved in the aircraft dynamics, the speed of sound $a$ and the aircraft Mach number $M = V/a$ are convenient to use in the acoustic analyses. The speed of sound is obtained from

$$a = (\gamma_e RT)^{1/2} \quad (6)$$

The gradient of the speed of sound satisfies the relation

$$\frac{da}{dz} = \frac{\gamma_e R}{2a} \frac{dT}{dz}$$

The time derivative of $V = Ma$ may then be expressed as

$$\frac{dV}{dt_a} = a \frac{dM}{dt_a} + M \frac{dz_a}{dt_a} \frac{da}{dz}$$

$$= a\left(\frac{dM}{dt_a} + M^2 \sin \gamma \frac{da}{dz}\right) \quad (7)$$

Application in the program.- Inputs into the program include $\psi$, $\gamma$, and $M$ as functions of the time $t_a$. Equations (2) are integrated to obtain the aircraft trajectory. The time derivatives of $\psi$, $\gamma$, and $M$ (actually of the Mach angle $\mu = \sin^{-1}(1/M)$ are required later in the program for the ray-tube area calculation and, for this purpose, two Maneuver Options are provided. In the first option, the load factors $n_L$ and $n_T$ are additional inputs, and the time derivatives are calculated from equations (5) and (7). In the second option, the time derivatives are calculated directly within the program by differentiating the input data.

The purpose of including Maneuver Option 1 is to provide a more accurate calculation of the time derivatives in case the load factors are accurately known, as perhaps from accelerometer data from a flight test. In this option, the input data are redundant, and it is the responsibility of the operator to ensure that they are reasonably consistent.

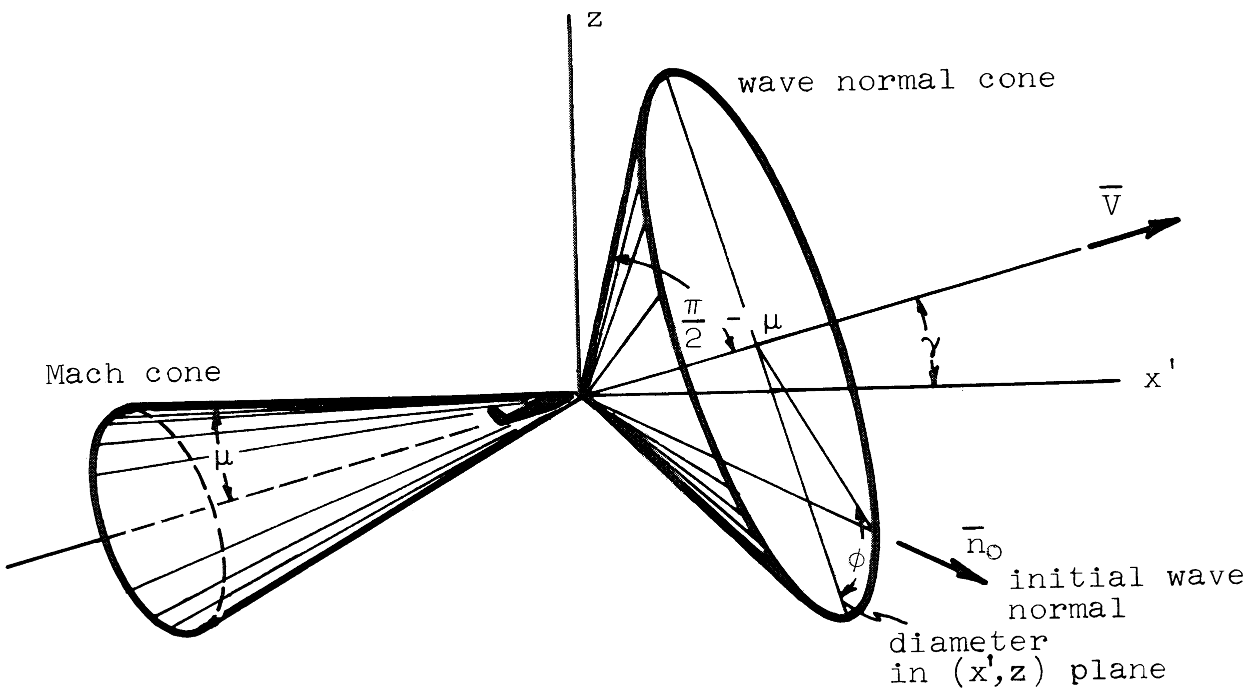
## Initial Wave Normals

The purpose of this section is to express the initial orientation of the wave fronts as they leave the aircraft. This initial orientation gives the basic parameters needed for the ray calculation.

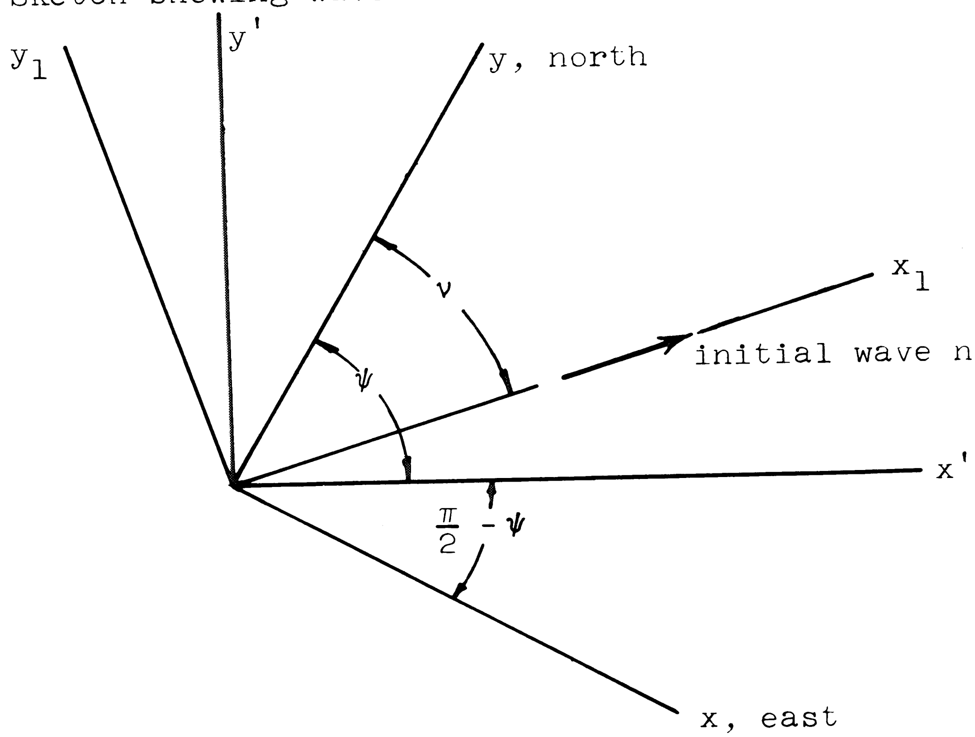
Here we have an example of the principle discussed earlier of using galilean invariance to identify the variables which are preferable for use in the analysis. The wave normals are the ray directions in one particular coordinate system, that fixed in the undisturbed atmosphere at the aircraft altitude. Ray directions are not galilean invariant, while wave front shapes and wave normals are. The use of wave normals rather than ray directions to define the basic variables keeps the analysis in its simplest form.

With each instant of time $t_a$ during the aircraft flight, we associate a Mach cone with vertex located on a given reference point on the aircraft. This Mach cone is tangent to the Mach conoid (wave front) moving with the aircraft. The normals to the Mach cone at the vertex form a wave normal cone. We consider an instantaneous coordinate system $(x',y',z)$ for purposes of describing direction only (fig. 1), rotated an angle $\pi/2 - \psi$ counterclockwise relative to the basic coordinate system, with origin at the reference point on the aircraft. The two cones are illustrated in this coordinate system in figure 3. The half-angle of the wave normal cone is the complement of the Mach angle $\mu$.

An arbitrary azimuth angle $\phi$ is chosen, measured according to a right-hand rule about the flight axis from the downward




(a) Sketch showing wave normal cone and one wave normal.




(b) Plan view showing projection of wave normal.

Figure 3. Geometry of initial wave normal directions, showing coordinate axis orientation.

direction (see figs. 2 and 3). The particular wave normal corresponding to this azimuth angle is considered and will determine the particular ray determined by the parameters $t_a$, $\phi$. The direction cosines ($\ell$,m,n) of the wave normal relative to the axes (x',y',-z) are to be determined next.

The pertinent vector directions may be represented by points on a unit sphere. The desired direction cosines can be derived either by spherical trigonometry or by calculating cartesian coordinates of points on the unit sphere. Choosing the first approach, we consider the spherical triangles of figure 4 and obtain

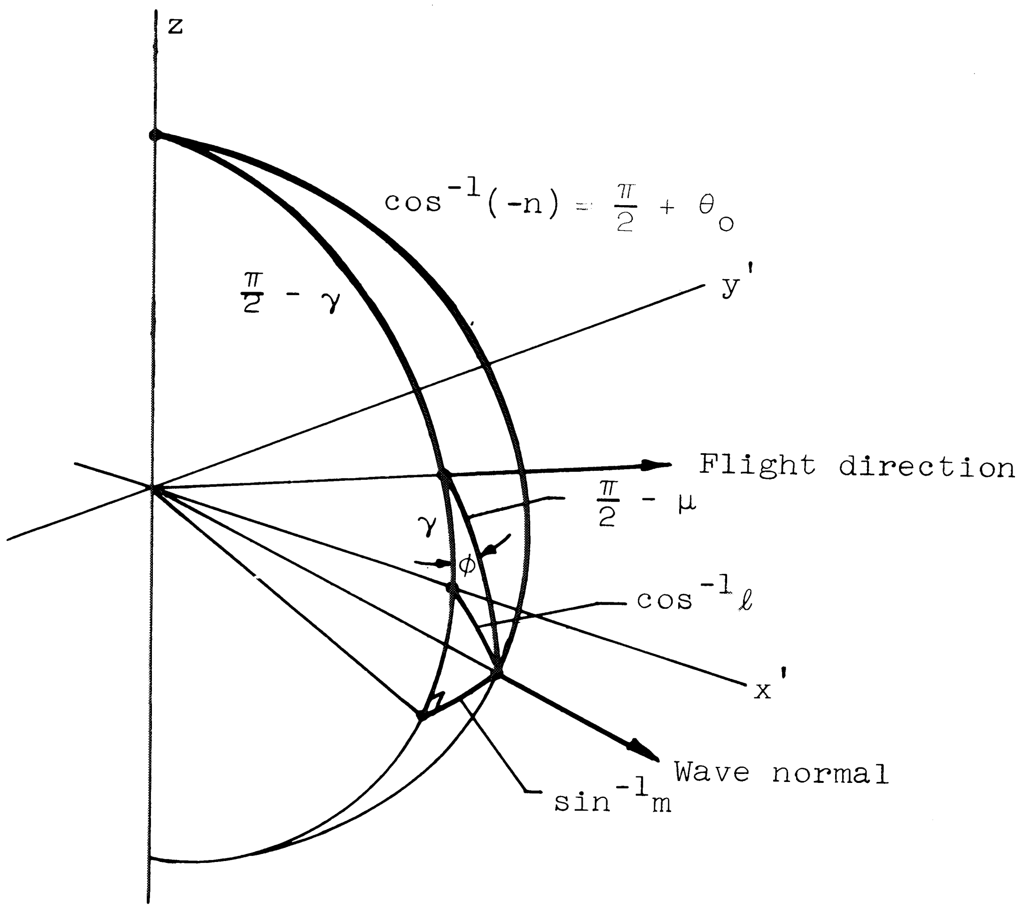



Figure 4. Spherical triangles for calculating wave normal direction cosines.

$$\ell = \sin\mu\cos\gamma + \cos\mu\sin\gamma\cos\phi$$

$$m = \cos\mu\sin\phi \qquad (8)$$

$$n = -\sin\mu\sin\gamma + \cos\mu\cos\gamma\cos\phi$$

using the law of cosines to obtain $\ell$ and $n$ and the law of sines for a right triangle to obtain $m$. Essentially the same results are obtained in Appendix IV of reference 3. The downward or $(-z)$ axis is used here because we want to consider primarily descending acoustic signals and rays.

The angle $\nu$ is defined as the heading angle of the wave normal (fig. 3(b)), and the angle $\theta_o$ is defined as the angle of the wave normal below the horizontal (fig. 4). Using these definitions, $m/\ell = \tan(\psi - \nu)$ and $n = \sin\theta_o$. With equations (8), these yield

$$\nu = \psi - \tan^{-1}\left\{\frac{\cos\mu\sin\phi}{\sin\mu\cos\gamma + \cos\mu\sin\gamma\cos\phi}\right\} \qquad (9)$$

and

$$\sin\theta_o = -\sin\mu\sin\gamma + \cos\mu\cos\gamma\cos\phi = n \qquad (10)$$

The maneuver history of the aircraft provides $\mu$ and $\gamma$, so that these equations give $\nu$ and $\theta_o$ as functions of the two ray parameters $t_a$ and $\ell$. We note that

$$\ell = \cos\theta_o\cos(\psi - \nu)$$

$$m = \cos\theta_o\sin(\psi - \nu)$$

Another coordinate system $(x_1, y_1, z)$ is shown in figure 3 and is aligned with a particular wave normal. This coordinate system is not used in this section but is used below in the treatment of ray tracing and ray-tube areas.

Application in the program.- Using the known values of $\psi$, $\gamma$, and $M$, the quantities $\nu$ and $\sin\theta_o$ are calculated from equations (9) and (10) as functions of the ray parameters $t_a$ and $\phi$.

## Mach Conoids and Ground Intersections

At this point we are ready to calculate the rays. The purpose of this section is to describe the rays and the Mach conoids (wave fronts) in general terms before going into the detailed calculation of the rays. The intent also is to show the functions needed to describe the rays and wave fronts globally and needed to determine when and where the sonic boom signals hit the ground. The primary purpose of the section is, thus, largely conceptual, and the reader primarily concerned with the algorithm may skip the section.

As the aircraft moves through space, a wave system associated with the aircraft moves with it and propagates away from the flight path. The wave system, which consists of a one-parameter family of wave fronts, is characterized by a single wave front chosen here to be the one of zero phase. This wave front is attached to the aircraft at the reference point on the aircraft and is tangent to the Mach cone associated with this point. The front is the same as the Mach cone only in the special case of straight flight at constant speed in an atmosphere of uniform temperature. This reference wave front is termed the reference Mach conoid and is shown schematically in figure 5.

The reference wave fronts or Mach conoids are not calculated directly. What we calculate are the kinematic rays corresponding to that wave front, which here are the rays of zero phase. With the basic assumption discussed in the section General Description, the ray paths for these rays are also those for other values of the phase and have been termed geometric rays.

The rays are specified by three functions giving $x$ , $y$ , and $z$ as functions of $t_a$ , $\phi$ , and $t$ . Here $t$ is the time on the ray, while $t_a$ and $\phi$ are the ray parameters defined in the preceding section. In a stratified atmosphere we replace $t$ by $z$ as independent variable. The ray is then specified by the two functions $x(t_a,\phi,z)$ and $y(t_a,\phi,z)$ describing the ray path (or geometric ray), together with the function $t(t_a,\phi,z)$ giving time along the ray. The calculation of these functions is described in the following section. The ray paths for a given time $t_a$ of emission are illustrated schematically in figure 5, shown here as straight lines. In the general case, they are curved lines.

The ground is at $z = 0$ . The solid ground intersection curve in figure 5 is given by the functions $x(t_a,\phi,0)$ and $y(t_a,\phi,0)$ for a given emission time $t_a$ . The time of arrival of the signal on this ground intersection curve is not constant and is given by the function $t(t_a,\phi,0)$ .

The wave fronts or Mach conoids are surfaces of constant $t$ . An inversion of the function $t(t_a,\phi,z)$ gives a function $t_a(\phi,z,t)$.

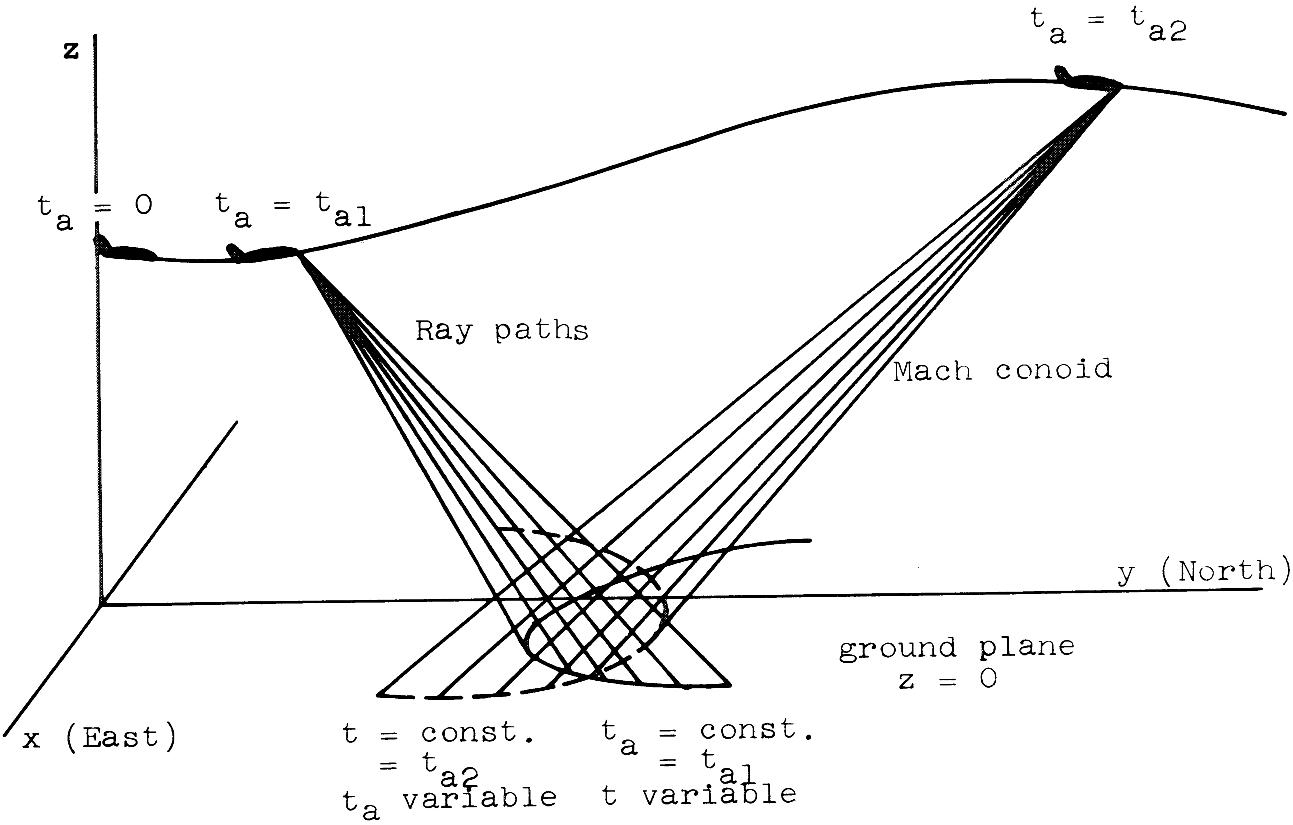



Figure 5. Maneuvering aircraft, showing coordinate system, rays, and Mach conoid.

When this is substituted into the ray path functions, the result is two functions, $x(\phi,z,t)$ and $y(\phi,z,t)$, giving the Mach conoids parametrized by the variables $\phi$ and $z$.

The intersection of a Mach conoid with the ground is the curve given by the functions $x(\phi,0,t)$ and $y(\phi,0,t)$, obtainable by the process just described but with $z$ kept equal to zero. Such a ground intersection of coincident signals is illustrated by the dashed curve in figure 5.

The phase variable which is introduced later in the analysis is a time $\xi$ measured from passage of the zero-phase wave front. Rays and wave fronts of phase different from zero obey the same equations as do those of zero phase, except that the arrival time $t$ is replaced by the time $t + \xi$.

<u>Application in the program</u>.- After the rays are calculated, following the procedure described in the next section, the Mach conoid ground intersections are obtained by interpolation of the

variable $t$. These ground intersection curves, together with corresponding maximum values of the pressure in the final signature, are part of the output.

## Snell's Law and Ray Tracing

The purpose of this section is to present the appropriate Snell's law for geometric acoustic propagation in a stratified atmosphere and to derive with this law the equations whose quadrature gives the rays.

In acoustic theory a signal on acoustic disturbance is propagated on a moving wave front. (From the mathematical point of view, a wave front is a characteristic hypersurface in four-dimensional space-time for the full equations of motion.) It moves in such a way that its normal velocity relative to the medium is the speed of sound. Its actual normal velocity in space is

$$c_n = a + \bar{n} \cdot \bar{u} \tag{11}$$

where $\bar{n}$ is a unit vector normal to the surface pointing in the direction of propagation and $\bar{u}$ is the vector velocity of the undisturbed medium (wind vector).

A signal initiated at one instant from a point is found a short time $\delta t$ later within a spherelet of radius $a\delta t$ whose center is displaced $\bar{u}\delta t$ from the original point. If every point on a wave front emits a signal at a given instant, Huygen's principle identifies one of the two envelopes of the spherelets $\delta t$ later as the wave front at that time (the other envelope corresponds to $-\bar{n}$ and is usually without meaning). This principle gives a motion to the wave front in accord with equation (11).

In geometric acoustics, the concept of a ray is fundamental. A ray is a point trajectory and may be defined

(a) as a characteristic in an asymptotic development of the equations of motion for small wave length;

(b) as a bicharacteristic for the full equations of motion, corresponding to the wave front as a characteristic hypersurface;

(c) to move from the point of emission of a spherelet to the point of tangency of the spherelet with the envelope wave front at a time $\delta t$ later.

Any of these definitions leads to the result that the ray is a trajectory of a point that moves with the velocity

$$\frac{d\bar{r}}{dt} = \bar{c} = a\bar{n} + \bar{u} \tag{12}$$

where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ is the position vector of the point and $\bar{i}$ , $\bar{j}$ , and $\bar{k}$ are cartesian unit vectors. This velocity is termed the ray velocity or group velocity. In geometric acoustics, the signal is propagated along rays with this velocity. Note that $c_n = \bar{n} \cdot \bar{c}$ .

Besides the ray or group velocity, an important entity is the slowness vector or inverse phase velocity $\bar{n}/c_n$ . This entity is more familiar in geometric optics than in geometric acoustics, primarily because the subject of geometric optics has been so thoroughly studied and applied to practical problems.

In order to calculate rays, it is necessary to know how the wave normal vector $\bar{n}$ changes along rays. A general refraction law may be derived (see ref. 4, for example) which states that

$$\frac{d\bar{n}}{dt} = \nabla a + (\nabla\bar{u}) \cdot \bar{n} - \bar{n}[\bar{n} \cdot \nabla a + \bar{n} \cdot (\nabla\bar{u}) \cdot \bar{n}] \tag{13}$$

along a ray. The combination of equations (12) and (13) is a system of differential equations which must be solved to obtain the ray.

In the case treated herein of a steady horizontally stratified atmosphere, the calculation of a ray is much simpler. A particular refraction law or Snell's law is available which gives $\bar{n}$ explicitly along a ray (see ref. 5, for example). This "Snell's law", stated in its most general form, is that the horizontal vector component of the inverse phase velocity vector $\bar{n}/c_n$ is constant along each ray. We decompose $\bar{n}$ into horizontal and vertical components according to

$$\bar{n} = \cos\theta\,\bar{n}' - \sin\theta\,\bar{k}$$

where $\bar{n}'$ is a horizontal unit vector. The angle $\theta$ is the angle of $\bar{n}$ below the horizontal. Our Snell's law then states that the horizontal vector $\cos\theta\,\bar{n}'/c_n$ is constant along each ray (see figs. 6 and 7). We define the velocity $c_o$ as

$$c_o = \frac{c_n}{\cos\theta} \tag{14}$$

in terms of which the invariant horizontal vector is $c_o^{-1}\bar{n}'$ . The initial value of $\theta$ when the ray is emitted from the aircraft is

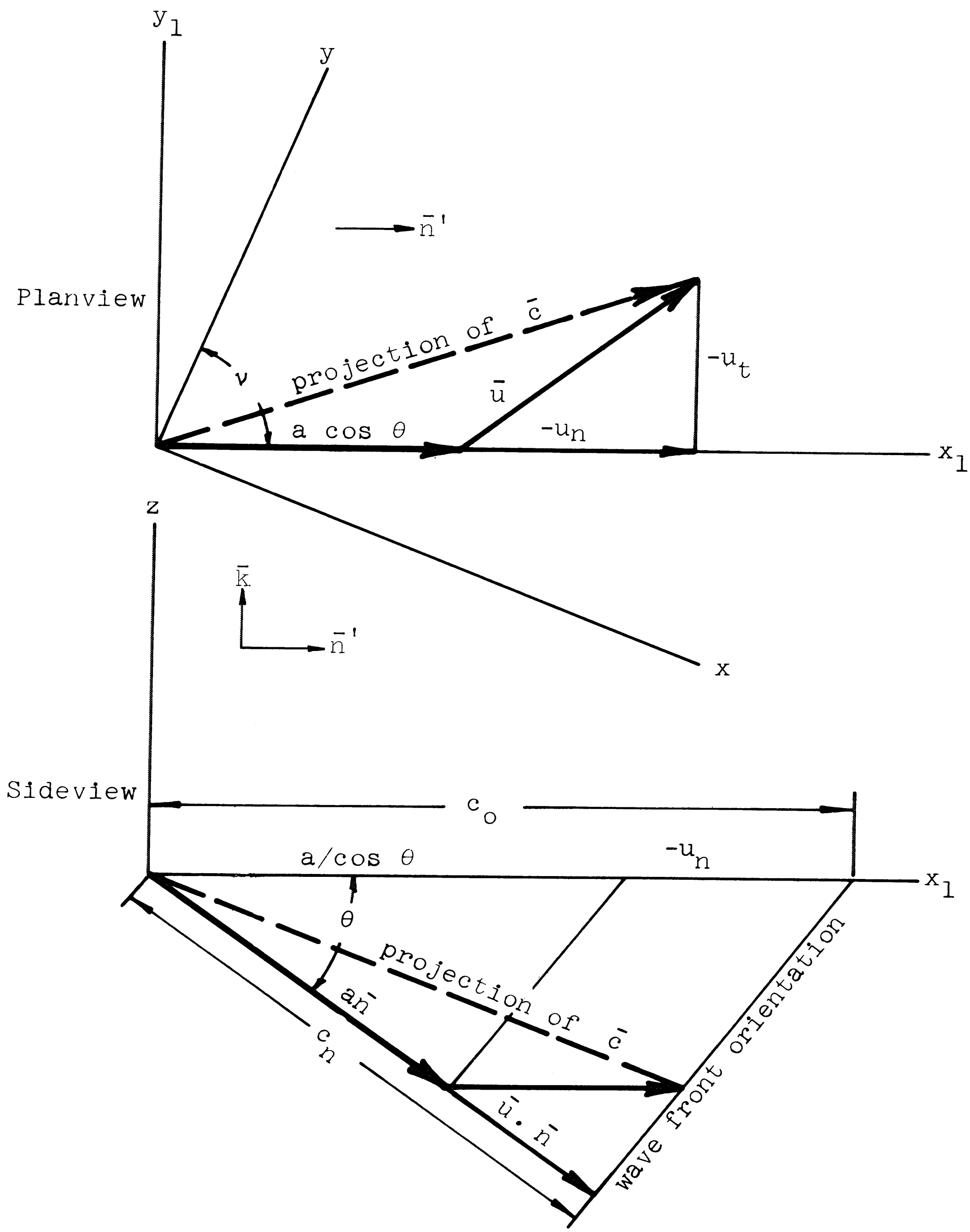



Figure 6. Propagation velocity plot.

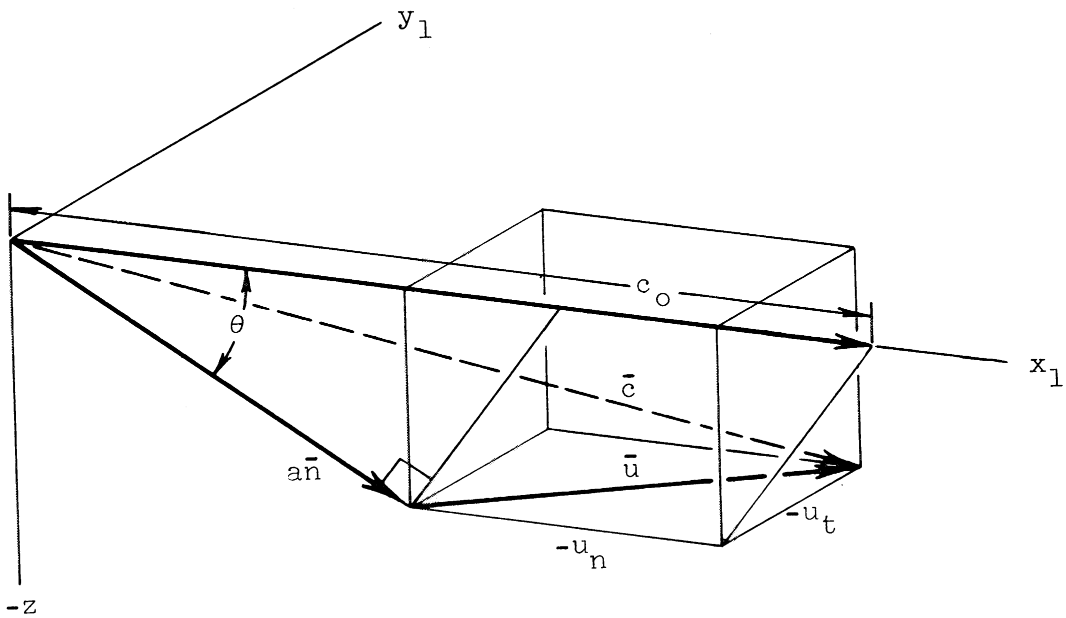



Figure 7. Oblique view of velocity plot.

the angle $\theta_o$ defined by equation (10). The quantity $c_o$ and the direction of $\bar{n}'$ are thus invariant along each ray. The direction or heading angle of $\bar{n}'$ is denoted $\nu$ in accord with the notation of the previous section. The two invariants $c_o$ and $\nu$ are functions $c_o(t_a,\phi)$ and $\nu(t_a,\phi)$ of the two ray parameters.

We introduce a coordinate system $(x_1,y_1,-z)$ aligned so that the wave normal $\bar{n}$ lies in the $(x_1,-z)$ plane (see fig. 3(b)). This system is one rotated an angle $\frac{1}{2}\pi - \nu$ counterclockwise relative to the basic coordinate system. The main use of this coordinate system will be in the following section, in the calculation of ray-tube area.

The wind vector $\bar{u}$ has components $(-u_x,-u_y)$ in the $(x,y)$ frame and components $(-u_n,-u_t)$ in the $(x_1,y_1)$ frame with

$$\begin{aligned} u_n &= u\cos(\nu-\eta) = u_x\sin\nu + u_y\cos\nu \\ u_t &= u\sin(\nu-\eta) = -u_x\cos\nu + u_y\sin\nu \end{aligned} \tag{15}$$

The minus signs before the components come from the ancient wind convention mentioned earlier. Note that $\bar{u}\cdot\bar{n} = -u_n\cos\theta$ .

For each initial wave normal (and corresponding value of $\phi$ ) in the wave normal cone at the aircraft, there is one corresponding ray. The value of $\nu$ for this ray is that obtained from equation (9). We obtain $c_o$ for the ray from the expression

$$c_o = \frac{c_{n_o}}{\cos \theta_a} = \frac{a_o + \bar{u}_o \cdot \bar{n}_o}{\cos \theta_o} = \frac{a_o}{\cos \theta_o} - u_{n_o} \tag{16}$$

where $\cos \theta_o$ is obtained through equation (10) and the subscript o denotes conditions at the initiation of the ray at the aircraft. At any other altitude, with $c_o$ and $\nu$ known, the angle $\theta$ may be calculated from

$$\cos \theta = \frac{a(z)}{c_o + u_n(z)} \tag{17}$$

Thus $\theta$ is known as a function of $c_o$ , $\nu$ , and z and thereby as a function of $t_a$ , $\phi$ , and z . Figures 6 and 7 show the relation between $c_o$ , $\theta$ , and a and the wind components $-u_n$ and $-u_t$ . The wave front appears edge-on in the $(x_1,z)$ plot. Note that the wind component $-u_t$ tangential to the wave front does affect $\bar{c}$ but has no effect on the Snell's law.

In order to carry out the ray tracing (to calculate the rays), we need to integrate the following equations (see figs. 6 and 7):

$$\frac{dx_1}{dt} = a \cos \theta - u_n$$

$$\frac{dy_1}{dt} = -u_t$$

$$\frac{d(-z)}{dt} = a \sin \theta$$

These equations are components of the vector equation (12). The independent variable is changed from t to -z , giving the equations

$$\frac{dx_1}{d(-z)} = \frac{a \cos \theta - u_n}{a \sin \theta}$$

$$\frac{dy_1}{d(-z)} = \frac{-u_t}{a \sin \theta}$$

A transformation from $(x_1,y_1)$ to $(x,y)$ gives the desired ray equations

$$\frac{dx}{d(-z)} = \frac{a \cos\theta \sin\nu - u_x}{a \sin\theta}$$

$$\frac{dy}{d(-z)} = \frac{a \cos\theta \cos\nu - u_y}{a \sin\theta} \tag{18}$$

$$\frac{dt}{d(-z)} = \frac{1}{a \sin\theta}$$

With a downward propagating ray $(\sin\theta > 0)$, the integration of equations (18) is carried out in the $-z$ direction from the initial point $(z = z_a, x = x_a, y = y_a, t = t_a)$ to the ground $(z = 0)$. The functions $\sin\theta$ and $\cos\theta$ are obtained as functions of $z$ from equation (17).

Equations (18) are the basic equations of this section as they yield the rays by quadratures. By integrating them we obtain the functions $x(t_a,\phi,z)$, $y(t_a,\phi,z)$, and $t(t_a,\phi,z)$ discussed in the preceding section. The equations immediately preceding equations (18) (in terms of $x_1$ and $y_1$) may be expressed with a minor modification to take neighboring rays into account. The ray equations in this form are more convenient than equations (18) for computing ray-tube areas and will be used for this purpose.

Application in the program.- In the program, for the selected values of the ray parameters $(t_a,\phi)$, $c_o$ is calculated from equation (16) with $u_{no}$ obtained from equation (15). Equations (18) are then integrated for the rays with $\theta$ obtained from equation (17). Only downward propagating rays are calculated, and the calculation is stopped when the ray is approximately horizontal.

Historical note.- The refraction law of the type of Snell's law, equation (14) or (17), was given by Lord Rayleigh in 1878 (Sect. 289 of ref. 6) in planar flow. Rayleigh did not distinguish between wave normals and rays, however. Barton, in 1901 (ref. 7), noted that the correct ray propagation velocity was not in the wave normal direction. He gave the correct planar ray tracing equations with examples. Fujiwhara, in 1912 (ref. 8), gave the correct Snell's law in three dimensions and the corresponding ray tracing equations (18) with examples. Emden (ref. 9) identified the ray in terms of energy transport without defining the energy.

## Ray-Tube Area

The purpose of this section is to obtain an expression for ray-tube area along the rays. The ray-tube area is needed subsequently in the analysis in order to express the acoustic signal quantitatively, and also thereby to calculate the nonlinear distortion.

A ray tube is a differential concept, to be visualized as a tube made up of rays which are differentially close to the particular ray being investigated. The actual cross-sectional area of such a tube is thus also a differential quantity. The quantity which we term ray-tube area is a finite measure of such a differential area and not actually a physically identifiable area. Multiplying any ray-tube area by a constant factor will make no difference in the final results of the analysis. A consequence of this fact is that the dimensions assigned to the ray-tube area are completely unimportant and may be changed through such a factor to suit our convenience. The invariance of the final result to a multiplication of the ray-tube area by an arbitrary constant factor was used as a check of the analysis.

In our case, we use the $(x_1, y_1, -z)$ coordinate system introduced earlier, corresponding to the particular angle $\nu = \nu_r$ for the reference ray, with the rays parametrized in terms of the ray parameters $t_a$ and $\phi$. Our use of Snell's law for a stratified atmosphere directs the use of horizontal cutting planes with the vector ray-tube area directed in the direction of the $-z$ axis. We can visualize a differential ray-tube area as the quadrilateral area on a plane $z$ = constant, determined by the rays with parameters $(t_a, \phi)$, $(t_a + \delta t_a, \phi)$, $(t_a, \phi + \delta\phi)$, and $(t_a + \delta t_a, \phi + \delta\phi)$ as illustrated in figure 8. This differential area is $\delta t_a \delta\phi$ times the Jacobian of $(x_1, y_1)$ with respect to $(t_a, \phi)$. We define the ray-tube area $A$ as

$$A(t_a, \phi, z) = \frac{1}{c_o} \begin{vmatrix} \dfrac{\partial x_1}{\partial t_a} & \dfrac{\partial y_1}{\partial t_a} \\ \\ \dfrac{\partial x_1}{\partial \phi} & \dfrac{\partial y_1}{\partial \phi} \end{vmatrix} \tag{19}$$

in terms of the functions $x_1(t_a, \phi, z)$ and $y_1(t_a, \phi, z)$. These we may conceive of as obtained by a rotation of $\frac{1}{2}\pi - \nu_r$ from the functions $x(t_a, \phi, z)$ and $y(t_a, \phi, z)$ that were obtained from integrating equations (1 ). The analysis is much simpler in this form. The factor $c_o^{-1}$ is included in the definition of $A$ so as

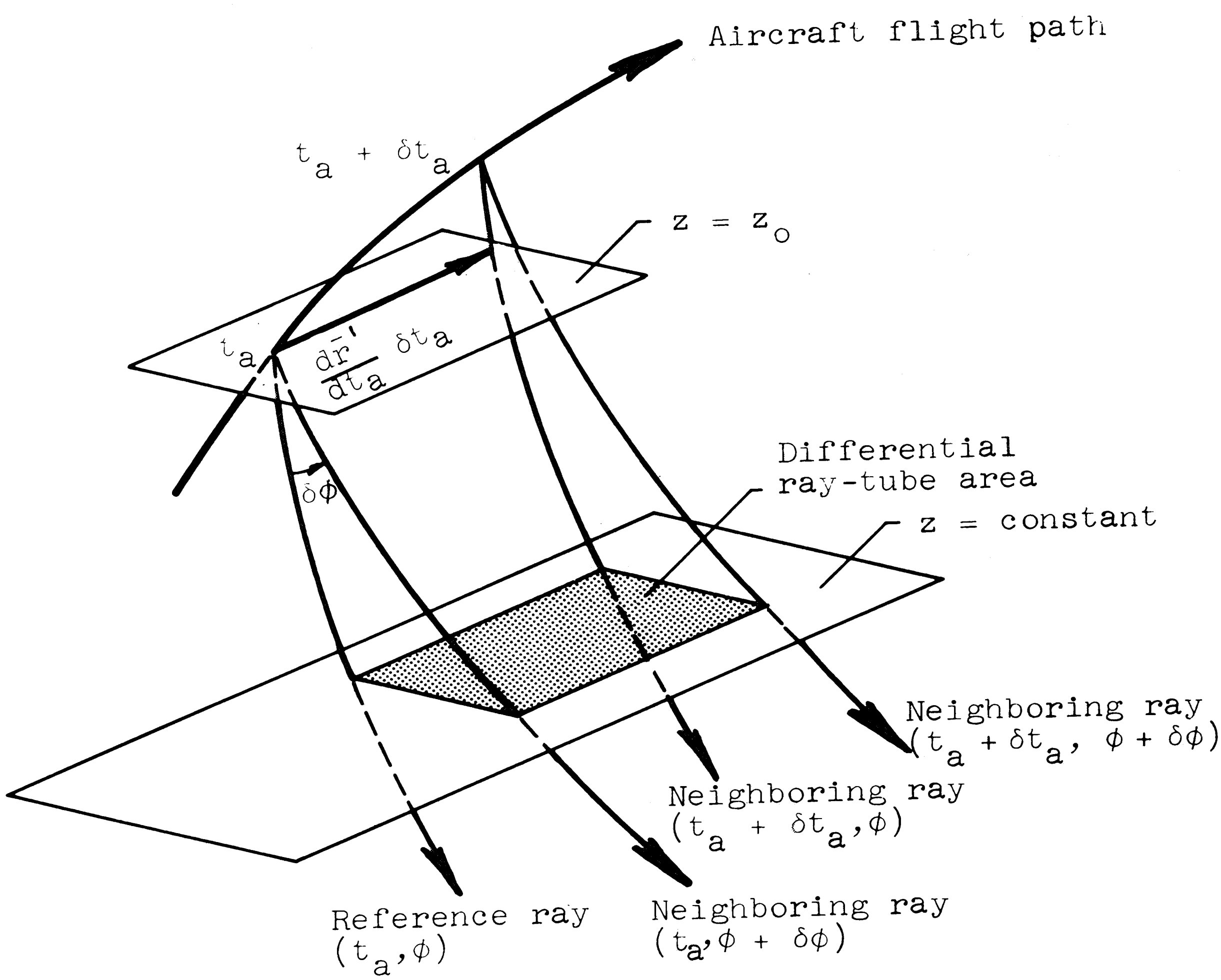



Figure 8. Sketch showing ray-tube area.

to make the subsequent formulas simpler; the factor $c_o^{-1}$ here, incidentally, makes A galilean invariant. Note that A has dimensions of length in this definition. We shall evaluate (19) by expressing the terms in the Jacobian as integrals taken downward along the ray and eventually arrive at the expression (26) below.

A few authors have defined entities analogous to ray-tube area in terms of Jacobians. In the acoustic stratified case, an identification equivalent to that of equation (19) was given by Lighthill (ref. 10). The recognition that the area may be obtained by quadratures and the area calculation in these terms is new in the present analysis. This treatment of ray-tube areas, developed in

somewhat more general terms, may also be found in reference 4. (Our analysis may be considered a special case of a general approach to calculating Jacobians of analogous type for general wave propagation; a paper on this subject by the first author is being prepared.)

The terms in the Jacobian of equation (19) are to be expressed as initial values plus integrals along the ray from $z = z_o = z_a$. The initial values (at the point of emission of the rays) of the $\phi$ derivatives are zero. The initial values of $\partial x_1/\partial t_a$ and $\partial y_1/\partial t_a$ are not zero. They form a horizontal vector $\partial \bar{r}'/\partial t_a$ (shown in fig. 8 multiplied by $\delta t_a$) where

$$\bar{r}' = x\bar{i} + y\bar{j} = x_1\bar{n}' + y_1\bar{k} \times \bar{n}'$$

We rewrite equation (19) in the form

$$c_oA = \begin{vmatrix} \left(\dfrac{\partial x_1}{\partial t_a}\right)_o & \left(\dfrac{\partial y_1}{\partial t_a}\right)_o \\ \dfrac{\partial x_1}{\partial \phi} & \dfrac{\partial y_1}{\partial \phi} \end{vmatrix} + \begin{vmatrix} \dfrac{\partial x_1}{\partial t_a} - \left(\dfrac{\partial x_1}{\partial t_a}\right)_o & \dfrac{\partial y_1}{\partial t_a} - \left(\dfrac{\partial y_1}{\partial t_a}\right)_o \\ \dfrac{\partial x_1}{\partial \phi} & \dfrac{\partial y_1}{\partial \phi} \end{vmatrix}$$

with the lower terms in the first determinant and all those in the second equal to zero at the point of emission. The variables $x_1$ and $y_1$ are expressible as integrals over $-z$, and their derivatives with respect to the ray parameters may be similarly expressed by differentiating under the integral sign. Thus, all the terms except the upper terms in the first determinant are integrals over $-z$ taken from the point of emission. We make a transformation of the independent variables $(t_a,\phi)$ to $(c_o,\nu)$ and write

$$c_oA = \begin{vmatrix} 0 & \left(\dfrac{\partial x_1}{\partial t_a}\right)_o & \left(\dfrac{\partial y_1}{\partial t_a}\right)_o \\ -\dfrac{\partial \nu}{\partial \phi} & \dfrac{\partial x_1}{\partial c_o} & \dfrac{\partial y_1}{\partial c_o} \\ \dfrac{\partial c_o}{\partial \phi} & \dfrac{\partial x_1}{\partial \nu} & \dfrac{\partial y_1}{\partial \nu} \end{vmatrix} + \begin{vmatrix} \dfrac{\partial c_o}{\partial t_a} & \dfrac{\partial \nu}{\partial t_a} \\ \dfrac{\partial c_o}{\partial \phi} & \dfrac{\partial \nu}{\partial \phi} \end{vmatrix} \begin{vmatrix} \dfrac{\partial x_1}{\partial c_o} & \dfrac{\partial y_1}{\partial c_o} \\ \dfrac{\partial x_1}{\partial \nu} & \dfrac{\partial y_1}{\partial \nu} \end{vmatrix} \qquad (20)$$

Here we have used the theorem that the determinant of the product of two matrices is the product of their determinants. The derivatives of $x_1$ and $y_1$ with respect to $c_o$ and $\nu$ in equation (20) are again integrals over $-z$ taken from the point of emission. Our next step is to evaluate these quantities.

The initial values of the $t_a$ derivatives may be calculated with the aid of the equations for the direction cosines of the initial wave normals. We note that

$$\ell = \cos \theta_o \cos (\psi - \nu)$$

$$m = \cos \theta_o \sin (\psi - \nu)$$

The result of the calculation is

$$\begin{aligned}
\left(\frac{\partial x_1}{\partial t_a}\right)_o &= V[\cos \gamma \cos(\psi - \nu) + \sin \gamma \cos \theta_o] \\
&\quad - u_{n_o}\left(1 + \frac{\sin \gamma}{\sin \mu \sin \theta_o}\right) \\
&= c_o\left(1 + \frac{\sin \gamma}{\sin \mu \sin \theta_o}\right)
\end{aligned} \tag{21}$$

$$\begin{aligned}
\left(\frac{\partial y_1}{\partial t_a}\right)_o &= - V \cos \gamma \sin (\psi - \nu) - u_{t_o}\left(1 + \frac{\sin \gamma}{\sin \mu \sin \theta_o}\right) \\
&= - \frac{V \cos \gamma \cos \mu \sin \phi}{\cos \theta_o} - u_{t_o}\left(1 + \frac{\sin \gamma}{\sin \mu \sin \theta_o}\right)
\end{aligned} \tag{22}$$

To calculate the derivatives of $(x_1, y_1)$ relative to $c_o$ and $\nu$, we first recognize that the coordinate system is defined to correspond to one reference ray with $\nu = \nu_r$ and that we must consider neighboring rays. The ray tracing equations (see equations preceding (18)) are written in terms of $x_1$ and $y_1$ and are

$$\frac{dx_1}{d(-z)} = \frac{\cos \theta}{\sin \theta} - \frac{u_n}{a \sin \theta}$$

$$\frac{dy_1}{d(-z)} = - \frac{\cos \theta}{\sin \theta}(\nu - \nu_r) - \frac{u_t}{a \sin \theta}$$

In the second equation, $(\nu - \nu_r)$ represents $\sin(\nu - \nu_r)$ with $\nu - \nu_r$ small. A factor $\cos(\nu - \nu_r)$ in the first equation has been set equal to 1. In the same terms, equation (17) relating $c_o$ and $\theta$ may be written

$$c_o = \frac{a}{\cos\theta} - u_n + u_t\,(\nu - \nu_r) \tag{23}$$

These three equations are differentiated at constant $z$ with respect to the three variables $c_o$, $\nu$, and $\theta$, and then $\nu$ is set equal to $\nu_r$. The differential of $\theta$ is eliminated to yield

$$\frac{\partial \dfrac{dx_1}{d(-z)}}{\partial c_o} = \frac{d\,\dfrac{\partial x_1}{\partial c_o}}{d(-z)} = -c_o\,\frac{\cos^3\theta}{a^2\sin^3\theta}$$

$$\frac{\partial \dfrac{dx_1}{d(-z)}}{\partial \nu} = \frac{d\,\dfrac{\partial x_1}{\partial \nu}}{d(-z)} = c_o\,\frac{u_t\cos^3\theta}{a^2\sin^3\theta}$$

$$\frac{\partial \dfrac{dy_1}{d(-z)}}{\partial c_o} = \frac{d\,\dfrac{\partial y_1}{\partial c_o}}{d(-z)} = \frac{u_t\cos^3\theta}{a^2\sin^3\theta}$$

$$\frac{\partial \dfrac{dy_1}{d(-z)}}{\partial \nu} = \frac{d\,\dfrac{\partial y_1}{\partial \nu}}{d(-z)} = -\frac{\cos\theta}{\sin\theta} - \frac{u_t^2\cos^3\theta}{a^2\sin^3\theta}$$

We define the following integrals

$$I_1(-z) = \int_{-z_o} \frac{\cos^3 \theta \, d(-z)}{a^2 \sin^3 \theta}$$

$$I_2(-z) = \int_{-z_o} \frac{u_t \cos^3 \theta \, d(-z)}{a^2 \sin^3 \theta} \tag{24}$$

$$I_3(-z) = \int_{-z_o} \left(\frac{u_t^2 \cos^3 \theta}{a^2 \sin^3 \theta} + \frac{\cos \theta}{\sin \theta}\right) d(-z)$$

as indefinite integrals equal to zero at $z = z_o$. This notation, avoiding the dummy variables of definite integrals, is a convenient one. In terms of these integrals, we can evaluate the quantities

$$\frac{1}{c_o} \frac{\partial x_1}{\partial c_o} = - I_1$$

$$\frac{1}{c_o} \frac{\partial x_1}{\partial \nu} = \frac{\partial y_1}{\partial c_o} = I_2 \tag{25}$$

$$\frac{\partial y_1}{\partial \nu} = - I_3$$

We now substitute the expressions obtained in equations (21), (22), and (25) into formula (20) for the ray-tube area. The result is

$$A = \left[\left(1 + \frac{\sin\gamma}{\sin\mu\,\sin\theta_o}\right)(I_2 - u_{t_o} I_1) - \frac{V\sin\phi\cos\mu\cos\gamma}{\cos\theta_o} I_1\right]\frac{\partial c_o}{\partial\phi}$$

$$- \left[\left(1 + \frac{\sin\gamma}{\sin\mu\,\sin\theta_o}\right)(I_3 - u_{t_o} I_2) - \frac{V\sin\phi\cos\mu\cos\gamma}{\cos\theta_o} I_2\right]\frac{\partial\nu}{\partial\phi}$$

$$+ \left(\frac{\partial c_o}{\partial t_a}\frac{\partial\nu}{\partial\phi} - \frac{\partial c_o}{\partial\phi}\frac{\partial\nu}{\partial t_a}\right)(I_1 I_3 - I_2^2) \tag{26}$$

This is the desired expression for $A$. This expression is given in terms of the derivatives of $c_o$ and $\nu$ with respect to the ray parameters, and these derivatives must now be calculated.

In the calculation of these derivatives, the quantities $\psi$, $\gamma$, and $\mu$ are functions of $t_a$ alone. The derivatives of $\nu$ are obtained with some algebraic manipulation from equation (9) and are

$$\frac{\partial\nu}{\partial t_a} = \frac{d\psi}{dt_a} + \frac{\sin\phi}{\cos^2\theta_o}\left(\cos\gamma\frac{d\mu}{dt_a} + \cos\mu\sin\theta_o\frac{d\gamma}{dt_a}\right)$$

$$\frac{\partial\nu}{\partial\phi} = -\frac{\cos\mu}{\cos^2\theta_o}(\cos\mu\sin\gamma + \sin\mu\cos\gamma\cos\phi) \tag{27}$$

To obtain the derivatives of $c_o$, we first differentiate equation (23) again at the aircraft (with $z$ variable) to obtain

$$dc_o = \frac{a_o\sin\theta_o}{\cos^2\theta_o}d\theta_o + u_{t_o}\,d\nu + \left(\frac{1}{\cos\theta_o}\frac{da_o}{dz} - \frac{du_{n_o}}{dz}\right)dz$$

Equation (10) for $\sin\theta_o$ is also differentiated, and the $\theta_o$ derivatives are eliminated. The quantity $\partial z/\partial\phi$ is zero and

$$\frac{\partial z}{\partial t_a} = \frac{dz_a}{dt_a} = \frac{a_o\sin\gamma}{\sin\mu}$$

We can then express the derivatives of $c_o$ as

$$\frac{\partial c_o}{\partial t_a} = u_{t_o}\frac{\partial \nu}{\partial t_a} - \frac{a_o \sin\theta_c}{\cos^3\theta_o}\left[(\cos\mu\sin\gamma + \sin\mu\cos\gamma\cos\phi)\frac{d\mu}{dt_a}\right.$$

$$\left. + (\sin\mu\cos\gamma + \cos\mu\sin\gamma\cos\phi)\frac{d\gamma}{dt_a}\right]$$

$$+ \frac{a_o \sin\gamma}{\sin\mu}\left(\frac{1}{\cos\theta_o}\frac{da_o}{dz} - \frac{du_{n_o}}{dz}\right) \tag{28}$$

$$\frac{\partial c_o}{\partial\phi} = u_{t_o}\frac{\partial\nu}{\partial\phi} - \frac{a_o\sin\theta_o\cos\mu\cos\gamma\sin\phi}{\cos^3\theta_o} \tag{29}$$

In equation (28) the derivative of $u_{n_o}$ with respect to $z$ is taken with $\nu$ constant in the fixed $(x_1, y_1)$ coordinate system. Thus, $du_{n_o}/dz$ is to be interpreted as $\sin\nu\, du_{x_o}/dz + \cos\nu\, du_{y_o}/dz$. Both $du_{n_o}/dz$ and $da_o/dz$ are, of course, evaluated at the aircraft altitude only.

Demonstration of the galilean invariance of $A$ is, of course, not essential to the analysis. Here we outline such a demonstration. The quantity $c_o$ is altered by an added constant in a galilean transformation in the $x_1$ direction, while a constant is added to the function $u_t(z)$ in a galilean transformation in the $y_1$ direction. However, the combination of terms

$$\frac{\partial c_o}{\partial t_a} - u_{t_o}\frac{\partial\nu}{\partial t_a}$$

and the corresponding quantity using $\phi$ derivatives are galilean invariant, as are the corresponding derivatives of $\nu$ alone. The combinations of integrals $I_2 - u_{t_o}I_1$ and $I_3 - 2u_{t_o}I_2 + u_{t_o}^2 I_1$

are also galilean invariant. From these results it may be shown from the expression (26) that $A$ is galilean invariant.

Application in the program.- For each ray, the derivatives of $c_o$ and $\nu$ relative to $t_a$ and $\phi$ are calculated from (27), (28), and (29) using information from the Aircraft Maneuvers Section. The integrals of equations (24) are computed along the ray. The ray-tube area $A$ is computed from equation (26) at the same time the ray is computed.

Historical note.- The concept of ray-tube area in sound propagation without planar, cylindrical, or spherical symmetry appears in a solution by Rayleigh in 1878 (Sect. 284 of ref. 6) with straight rays. Most discussions in the literature of ray-tube area with curved rays have been confined to cases in which the aircraft is in steady level flight (no dependence upon $t_a$); in these cases the solutions are much simpler than in the general case.

## Flow Near the Aircraft

The purpose of this section is to define the F-function used in the analysis and to present an outline of the local theory near the aircraft which leads to the concept of the F-function. The F-function is needed as an initial acoustic signal in the basic geometric acoustics calculation. The F-function can be directly computed by linear theory from the geometry and lift distributions of a slender aircraft. The reader uninterested in the details of this computation may skip to equation (33) where the F-function as used in this analysis is defined.

The initial conditions for the calculation of sonic boom propagation must be obtained from the flow field near the aircraft. This section reviews the local theory near the aircraft which leads to the concept of an F-function. This F-function is the function in terms of which initial conditions are specified.

Thus we describe here briefly the linear solution for the flow about an aircraft with particular attention to the outer asymptotic form of this solution. We assume, for simplicity, that the slender-body and thin-wing assumptions of linearized supersonic aerodynamic theory are valid, and that the aircraft may be represented by a combination of linear and surface distributions of source and lifting elements.

We assume further, presuming the shape of the aircraft is given, that the problem of finding the lift distributions has been solved. Thus we shall treat the lift distributions (and correspondingly the side force distributions) as known. With the slender

body assumption, the source distributions are obtainable directly from the aircraft shape. For simplicity in notation, we assume a single line distribution (fuselage or nacelle) and a single surface distribution (the wing). A complete aircraft will require several of each type.

For this section only, we use an $(x,y,z)$ coordinate system fixed with respect to the aircraft, with the undisturbed velocity $V$ in the direction of the $x$-axis. The $z$-axis is vertical; the $y$-axis is lateral. The velocity potential in the undisturbed flow is $Vx$; let $V\Phi$ be the perturbation to the velocity potential. The reduced perturbation potential $\Phi$, which has the dimensions of distance, may be divided into the part due to the line distribution and the part due to the surface distribution. Each of these may be divided into contributions from sources (representing cross-sectional area), from lift (z-forces), and from side force (y-forces).

The line distribution contribution from sources is

$$\Phi = \frac{-1}{2\pi} \int_{-\infty} \frac{A'(x_o)dx_o}{R}$$

where

$$R = [(x - x_o)^2 - \beta^2(y - y_o)^2 - \beta^2(z - z_o)^2]^{1/2}$$

$\beta^2 = M^2 - 1$ and $y = y_o$, $z = z_o$ is the axis of the line distribution; the integral is taken to the value of $x_o$ for which $R = 0$ to the upstream Mach cone from the point $(x,y,z)$. The lower limit $-\infty$ simply means far enough upstream to include all disturbances. The quantity $VA'$ is the linear source strength distribution; $A(x_o)$ represents the cross-sectional area of the body represented by the source distribution, and $A'(x_o)$ is its derivative.

The line distribution contribution from lift is

$$\Phi = \frac{1}{2\pi\rho V^2} \int_{-\infty} \frac{(z - z_o)(x - x_o)}{(y - y_o)^2 + (z - z_o)^2} \frac{f_z dx_o}{R}$$

and that from side force by the same expression with $(z - z_o)$ replaced by $(y - y_o)$ in the numerator of the integrand and with $f_z(x_o)$ replaced by $f_y(x_o)$. The distribution $f_z(x_o)$ is the lift force per unit distance on the axis of the line distribution.

Surface distributions of sources and lifting elements are distributions on a cylindrical mean surface $z = z_o(x_o,y_o)$. The contribution to $\Phi$ of the surface source distribution is of the form

$$\Phi = \frac{-1}{2\pi} \int\int_{-\infty} \frac{t'(x_o,y_o)\, dy_o\, dx_o}{R}$$

Here, $t(x_o,y_o)$ represents the wing thickness distribution measured in the z-direction, and $t'$ its derivative in the $x_o$ direction. The lift and side force contributions of the surface may be expressed analogously; here $f_y$ and $f_z$ are replaced by distributions of force per unit projected area with an integration over $y_o$.

We next pass to cylindrical coordinates $(x,r,\phi_r)$ with $r^2 = y^2 + z^2$ and $y = -r \sin \phi_r$, $z = -r \cos \phi_r$. With this definition, the ray parameter $\phi$ will be given by $\phi_a + \phi_r$ where $\phi_a$ is the bank angle. Our purpose is to define for each value of $\phi_r$ a body of revolution equivalent to the aircraft for a distant observer. We define further

$$r_o(y_o,z_o,\phi_r) = -y_o \sin \phi_r - z_o \cos \phi_r$$

$$s_o(y_o,z_o,\phi_r) = -y_o \cos \phi_r + z_o \sin \phi_r$$

In terms of these variables, we can write

$$R = [(x - x_o)^2 - \beta^2(r - r_o)^2 - \beta^2 s_o^2]^{1/2}$$

Far from the aircraft, with $x_o$ and $\beta r$ both large but with $x - \beta r$ not large, the term $s_o^2$ will be negligible. We drop this term and write

$$R = [x - x_o + \beta(r - r_o)]^{1/2} [x - x_o - \beta(r - r_o)]^{1/2}$$

We next neglect $x - \beta r - x_o - \beta r_o$ in comparison with $\beta r$ and write

$$R = (2\beta r)^{1/2}[x - \beta r - (x_o - \beta r_o)]^{1/2}$$

In the expression for the contribution due to lift, making the analogous asymptotic approximations, we replace the factor

$(z - z_o)(x - x_o)[(y - y_o)^2 + (z - z_o)^2]^{-1}$ by $-\beta \cos \phi_r$ and the analogous factor for the side force by $-\beta \sin \phi_r$. We also replace the dummy variable $x_o$ by

$$x_1 = x_o - \beta r_o$$

In the contributions from the surface distributions the integrals with respect to $y_o$ are taken with $x_1$ constant. We obtain thereby expressions in terms of new equivalent line distributions which are functions of $x_1$; thus, for example, $A_w$ is defined

$$A_w(x_1,\phi_r) = \int_{x_1=\text{const.}} t \, dy_o$$

Distributions $f_{yw}$ and $f_{zw}$ are defined analogously.

We now define an equivalent area distribution $S'(x_1,\phi_r)$ which replaces all the others, by the relation

$$S' = A'(x_1 + \beta r_o) + A_w'(x_1) + \frac{\beta}{\rho V^2} \ell(x_1)$$

where

$$\ell(x_1) = f_y(x_1 + \beta r_o)\cos \phi_r - f_z(x_1 + \beta r_o)\sin \phi_r$$

$$+ f_{yw} \cos \phi_r - f_{zw} \sin \phi_r$$

In these expressions $r_o$ is the value for the line distribution. Assembling the terms together gives for $\Phi$ the asymptotic expression

$$\Phi(x - \beta r,r,\phi_r) = \frac{-1}{2\pi(2\beta r)^{1/2}} \int_{-\infty}^{x-\beta r} \frac{S'(x_1)dx_1}{(x - \beta r - x_1)^{1/2}} \qquad (30)$$

The distribution $S'(x_1,\phi_r)$ is the area distribution of an equivalent body of revolution on the $x$ axis, as seen by an observer a large distance away in the direction $\phi_r$. The distribution $S'(x_1)$ is the sum of two terms. The first is $A' + A_w'$, the $x_1$ derivative of the projected cross-sectional area cut by the planes $x_1 = \underline{\text{const.}}$ The second is proportional to $\ell(x_1)$, the equivalent force distribution formed by the sum over planes $x_1 = \underline{\text{const.}}$ of force components in the direction opposite to $\phi_r$.

With $(x,y,z)$ fixed, the surface $R = 0$ is an upstream-facing Mach cone in the space $(x_o,y_o,z_o)$. This represents a surface of "coincident signals" for the point $(x,y,z)$. When such points are far from the aircraft, these surfaces are approximately the planes $x_1 = \underline{\text{const.}}$ (see fig. 9).

In an actual computation, the determination of the function $S'(x_1,\phi_r)$ may be fairly complicated. The force distributions must be obtained, of course. Interference contributions generally must be computed (ref. 11). If the slender-body approximation is inappropriate for any line distribution, additional analysis is needed to obtain the appropriate singularity distribution. The inlet captured area and the exit jet area must be included with the engine nacelle. Finally, the contributions from all line distributions and all surface distributions are combined.

With $S'$ assumed known, we differentiate equation (30) with respect to $x$. The perturbation pressure $\Delta p$ is given by $-\rho V^2\Phi_x$. The result of the differentiation is

$$-\Phi_x = \frac{1}{2} C_p = \frac{1}{M^2}\frac{\Delta p}{\rho a^2} = \frac{1}{M^2}\frac{q}{a} = \frac{1}{(2\beta r)^{1/2}} F_i(x - \beta r,\phi_r) \tag{31}$$

where

$$F_i(x - \beta r,\phi_r) = \frac{1}{2\pi}\int_{-\infty}^{x-\beta r} \frac{S''(x_1,\phi_r)\, dx_1}{(x - \beta r - x_1)^{1/2}} \tag{32}$$

Here we have introduced the magnitude of the perturbation velocity $q$ and the perturbation pressure $\Delta p = \rho a q$, variables which are appropriate for geometric acoustics. Equation (31) gives a solution which fits geometric acoustic theory. The $r$ which appears in the factor $(2\beta r)^{-1/2}$ is a ray-tube area, $x - \beta r$ is a phase, and $\phi_r$ and $x + \beta^{-1}r$ are ray parameters. In this case of steady flight, the solution is essentially independent of the second ray parameter (which will correspond to $t_a$). Figure 9 shows the wave front $x - \beta r = \underline{\text{const}}$. which are essentially the same family of planes as those determined by constant values of the dummy variable $x_1$.

Equation (32) defines an F-function in the way in which it is usually defined. For the purpose of this analysis, a somewhat different definition is used. In place of equation (31) we write

$$\frac{\Delta p}{\rho a^2} = \frac{q}{a} = \frac{1}{r^{1/2}} F \tag{33}$$

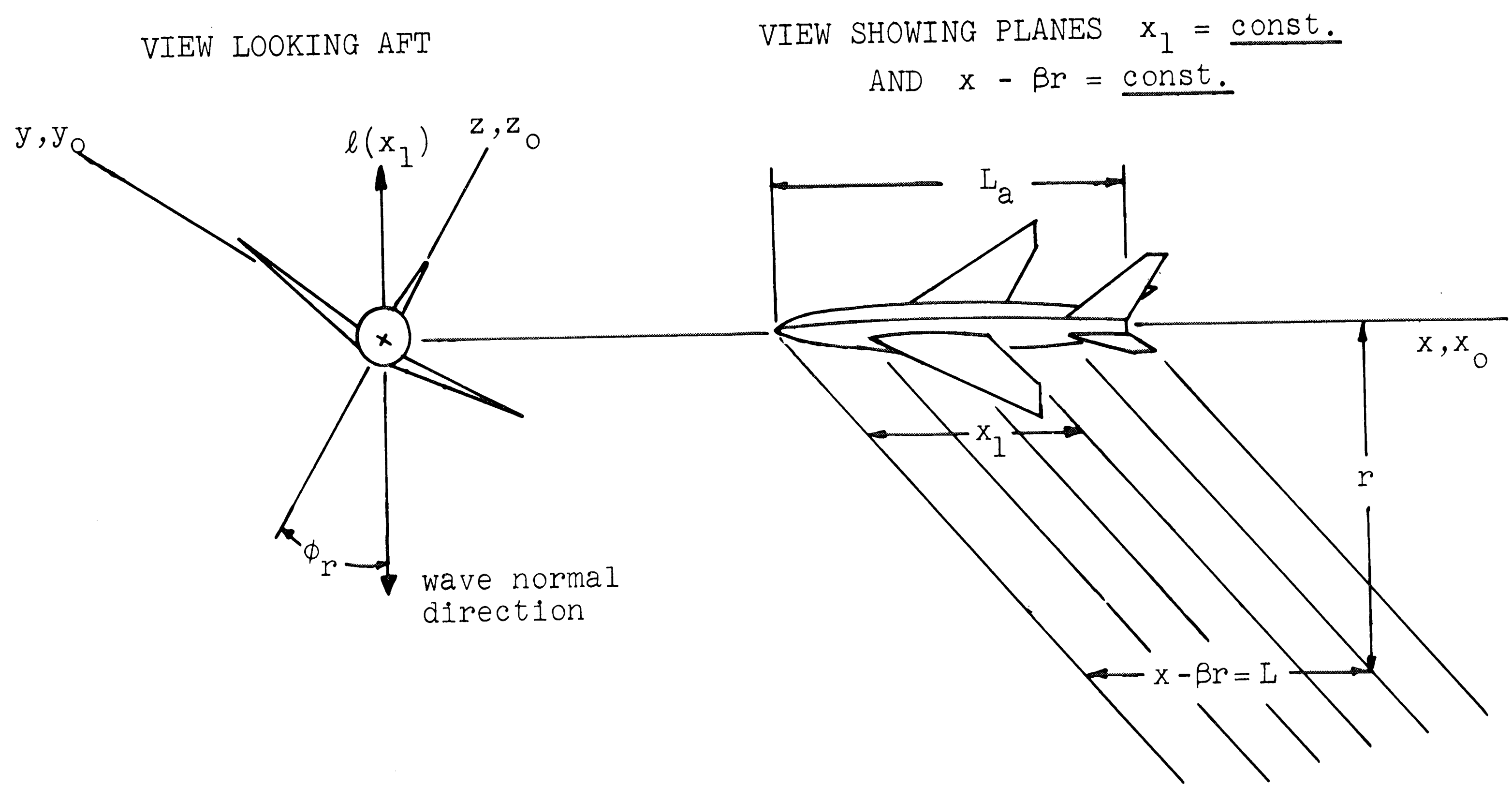



Figure 9. Local coordinate systems.

The phase variable $x - \beta r$ is denoted $L$, in terms of distance along the aircraft axis. This quantity is divided by $L_a$, the length of the aircraft, to form the dimensionless phase variable $L/L_a$. The F-functions are directly related, of course, and we write

$$F = F_f F_i \tag{34}$$

The quantity $F_f$ is simply a conversion factor, to convert any given F-function $F_i$ to the form corresponding to definition (33). With $F_i$ defined by equation (32), this factor is

$$F_f = \frac{M^2}{(2\beta)^{1/2}} \tag{35}$$

A plot of this particular function is shown on figure 10.

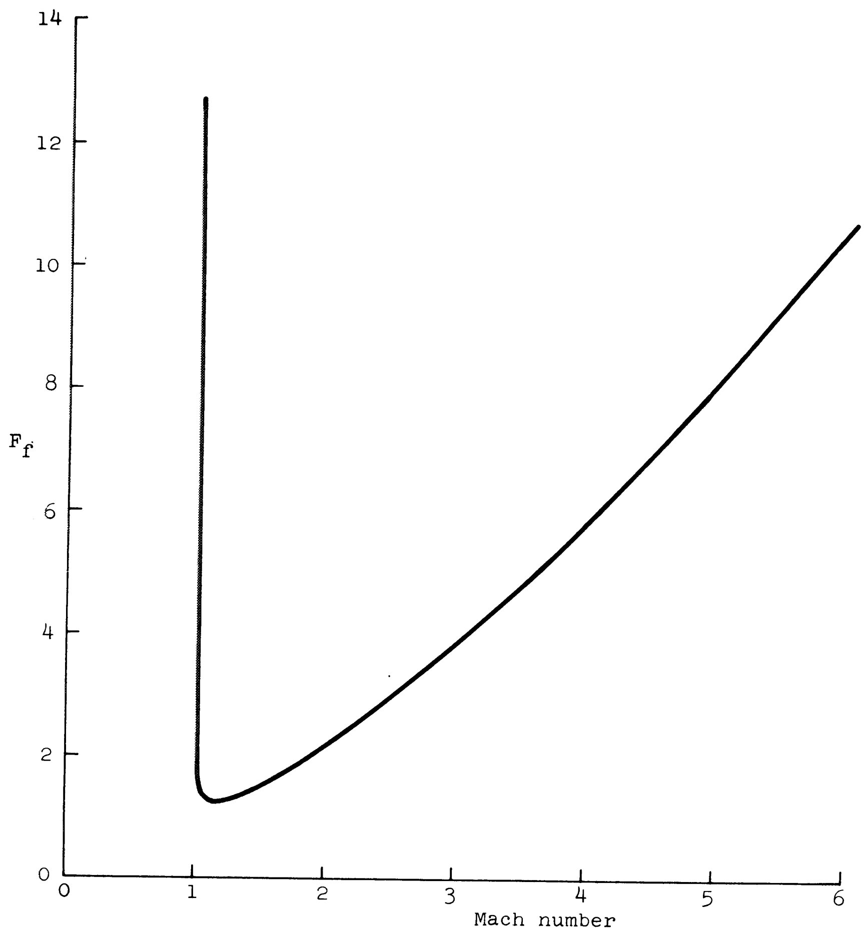



Figure 10. F-function factor.

Not important in our sonic boom analysis but basic to the linearized theoretical aerodynamics of the aircraft is the relation between the F-function and wave drag. The inviscid drag of the aircraft may be divided into two parts - the wave drag represented by energy in the wave system and the vortex drag represented by energy in the trailing vortex system. The wave drag may be represented by

$$C_D = \int_0^{2\pi} C_{D_{\phi_r}} \, d\phi_r \tag{36}$$

where

$$C_{D_{\phi_r}} = \frac{1}{S_{ref}} \int_{-\infty}^{\infty} F_i(L,\phi_r)^2 \, dL$$

in terms of the $F_i$ defined in equation (32). A transformation leads to the alternative expression

$$C_{D_{\phi_r}} = \frac{1}{8\pi^2 S_{ref}} \int_{-\infty}^{\infty} \int_{-\infty}^{x_o} S''(x_o) S''(x_1) \ln(x_o - x_1) \, dx_1 \, dx_o \tag{37}$$

Thus there is a direct relation between the F-function and the drag of the aircraft.

The F-function, as we have described it, is a function of the local phase $L/L_a$ (or $x - \beta r$) and of the azimuth angle $\phi_r$. It is also dependent upon the aerodynamic state of the aircraft (in dimensionless terms). This state is determined by the values of $M$ and $C_L$, the center of gravity location and the engine operating state, with some minor dependence upon other parameters (in a maneuver, for example). We include the dependence upon $M$ and $C_L$ in our notation, and write

$$F = F(L/L_a, \phi_r, M, C_L) \tag{38}$$

The quantities $M$ and $C_L$ are known functions of time $t_a$, as is the angle of bank $\phi_a = \phi - \phi_r$. Thus, in the remainder of the analysis, $F$ is considered a function $F(L/L_a, t_a, \phi)$ of phase and the two ray parameters.

The linear theory described in this section does not give the only method for obtaining F-functions. Any local aerodynamic analysis carried out to a large enough distance from the aircraft in the wave system will yield the F-functions. They may also be

obtained from wind tunnel tests or from flight tests, with perhaps minor modifications in the consequent analysis because of nonlinear effects.

For example, let us consider that pressure measurements are made with a microphone mounted under a balloon in an atmosphere without winds. The aircraft flies by at a distance $r$ in level flight. The pressure perturbation $\Delta p$ is measured as a function of the time $\xi$ from the appearance of the signal. The function $F$ is then obtained from equation (33). The phase argument of $F$ in terms of $L$ is obtained from $L = V\xi$. The nonlinear correction, if needed, may be carried out in two ways. One straightforward approach would be to consider the measured F-function to correspond to an acoustic signal that has already been distorted, to assign an initial value $\tau_o$ of the age (defined below in eq. (46)) to account for this initial distortion, and to proceed as in the next three sections with $\tau$ replaced by $\tau - \tau_o$. This approach would entail additional analysis to compute $\tau_o$. A simpler method is to apply the procedure of the next three sections to the experimental case in reverse, starting with the pressure signature and going to an F-function. In this reverse procedure, segments of the phase may appear in which $F$ is undefined, or rather is not uniquely defined. This nonuniqueness does not matter in applications for which the distortion is sufficiently great ($\tau - \tau_o > 0$). One correct way of filling the empty phase segments for the purpose of this analysis would be by connecting the known portions of the curve with straight line segments (with $F$ then continuous).

Application in the program.- The F-function must be specified as an input function $F_i(L/L_a, t_a, \phi)$. The conversion factor $F_f$ is specified or calculated as a function of $t_a$. The function $F$ is then obtained from equation (34) for use in the subsequent analysis.

Historical note.- The theory of this section was given by Hayes in 1947 (ref. 12), primarily as the basis of a method of computing wave drag by means of equations (36) and (37). An alternative approach was given by Lomax in 1955 (ref. 13; see also ref. 14). An exposition of sonic boom theory in a uniform atmosphere by Walkden in 1958 (ref. 15), based on an earlier paper of Whitham (ref. 16), also uses an F-function dependent upon $\phi_r$ as a parameter. The term F-function stems from a basic paper by Whitham (ref. 17).

## Geometric Acoustics and Blokhintsev Invariance

The purpose of this section is to define an appropriate invariant measure of the acoustic signal along each ray, one that is valid globally. This measure of the signal will be expressed in

terms of the F-function for the aircraft.

Geometric Acoustics is a linear acoustic theory based upon the assumption that the solution at every point in the field appears locally as it would in a plane-wave system. The wave fronts are actually gently curved surfaces, but locally they appear to be planar. The local solution, therefore, takes the form appropriate to a plane-wave system, with the perturbation velocity $q\bar{n}$ normal to the wave fronts and the perturbation pressure $\Delta p$ given by

$$\Delta p = \rho a q \tag{39}$$

As in a plane wave system, the quantity $q$ is a function of phase.

When the undisturbed flow field is steady in a specific inertial coordinate system, as in the case we are considering, the time required for a signal to be propagated between two fixed points is a constant. The pressure perturbation experienced at each of these points is a function of time measured from the passage of a reference signal. The function at the second point is proportional to the same function at the first point. This observation indicates that the time measured from the passage of a reference wave front is a suitable choice for phase. The reference wave front is then the wave front of zero phase. The signal phase measured from this zero reference is termed $\xi$, and in the following paragraphs will be related to the variable $L/L_a$ in terms of which F-functions are specified. An alternative demonstration that the time $\xi$ here defined is a suitable phase may be found in reference 4.

The fact that a directly defined, useful, physical entity $\xi$ is a suitable phase in this case is fortuitous and depends upon the restriction to steady atmospheric properties. This convenient element would not appear in an acoustic calculation in an unsteady atmosphere, where a phase variable with no direct physical interpretation would have to be used.

The ratio $q/a$ is used as a measure of the signal intensity. In a plane-wave system in a uniform atmosphere, this quantity would be a function only of phase, constant on each wave front and thus constant on each kinematic ray. With the atmosphere non-uniform and with the wave fronts curved, $q/a$ is no longer constant on each kinematic ray. In one approach to this intensity problem, the intensity is expressed as an integral along the ray (refs. 18 and 19). In this approach the integrand always includes a term proportional to the wave-front curvature, which must then be obtained from some other computation. The alternative approach

which we follow depends upon theoretically established invariant measures of the intensity. This approach requires the concept of ray-tube area.

In an atmosphere at rest, with no winds, volume integrals of the Rayleigh acoustic energy density (ref. 6, Sects. 245 and 295) are conserved. This acoustic energy density is $\rho q^2/2 + \Delta p^2/2\rho a^2$. Within geometric acoustics, with equation (39) valid, this energy density is simply $\rho q^2$. Half the energy is kinetic and half potential. The energy flow down a ray-tube is $\rho q^2 a A_n$, where $A_n$ is the normal ray-tube area, and is constant on a kinematic ray. Thus, if $q/a$ is known as a function of phase at one point on a geometric ray, it can be predicted at another. Rayleigh (ref. 6, Sects. 266 and 284) used essentially this approach to predict the effect of density and ray-tube area changes.

In an atmosphere with steady winds, an analogous invariance result was found by Blokhintsev in 1946 (Sect. 7 of ref. 20, or ref. 21), valid only within geometric acoustics. A straightforward derivation may be found in reference 22, Sect. 1. The invariant density is $\rho q^2 c_n/a$ and the quantity that is constant along a kinematic ray is this density times the volume flow $\bar{c} \cdot \bar{A}$. We express this constant quantity as $c_o$ times the square of a variable $V_E$ with the sign of $V_E$ the same as that of $q$. We have already defined $\bar{A}$ as $A$ times the unit vector in the $-z$ direction. The invariant is thus

$$c_o V_E^2 = \frac{\rho q^2}{a} A c_n a \sin\theta$$

or, using the relation $c_n = c_o \cos\theta$ (eq. 14),

$$V_E(\xi, t_a, \phi) = \frac{q}{a}\left(\rho a^2 A \sin\theta \cos\theta\right)^{1/2} \tag{40}$$

The purpose of the $c_o$ factor in the definition of $V_E^2$ is to make the subsequent analysis simpler and also to make $V_E$ galilean invariant. Once $V_E(\xi)$ is known for a particular geometric ray, we can solve directly for the pressure perturbation $\Delta p(\xi)$ as a function of the actual time $t + \xi$ using equations (39) and (40). Here $t$ is the time obtained in the ray tracing computation for the kinematic rays of zero phase.

At this point, we come to the critical step mentioned in the General Description of the analysis, in which a consideration of a galilean transformation is inescapable. The local solution is given as a function $F$ of $L/L_a$ and two ray parameters in a

coordinate system moving with the velocity of the aircraft. The global solution is of the form of a function $V_E$ of $\xi$ and the two ray parameters $t_a$ and $\phi$ in a coordinate system fixed relative to the ground. In order to express the solution in terms of $V_E(\xi)$, we must relate both the dependent and independent variables in the two coordinate systems. Although what has technically to be accomplished is the galilean transformation, much of the job has already been done by the identification of ray parameters and phase.

The ray parameters give us no trouble, as the galilean transformation has already been accomplished in the details of calculating the initial wave normals and of the ray tracing. The parameter $t_a$ is identified locally through the functions $M(t_a)$, $C_L(t_a)$, and $\phi_a(t_a)$. The local and global azimuth angles are connected through the relation $\phi = \phi_a + \phi_r$.

To relate the two phase variables, we use the stratagem of introducing a third, local, phase variable which is readily interpretable in either the local or ground-fixed coordinate system and has the property that it is galilean invariant. Such a variable is the distance $s$ normal to the wave fronts, measured from the reference wave front at a given instant. In the local coordinate system, we have

$$s = L/M = L \sin \mu$$

The quantity $L$ is distance from the reference wave front in a direction $\frac{1}{2}\pi - \mu$ from the normal. In terms of the phase $\xi$ introduced in this section, the distance $s$ is

$$s = \xi c_{n_o} = \xi c_o \cos \theta_o$$

We equate the two expressions for $s$ and obtain the basic phase relation

$$\xi = \frac{L_a \sin \mu}{c_o \cos \theta_o} \frac{L}{L_a} \tag{41}$$

The presence of the factor $c_o$ in this relation indicates that $\xi$ is not a galilean invariant quantity.

To relate the dependent variables $V_E$ and $F$, we follow an analogous course. The dimensionless measure $q/a$ of the signal is clearly galilean invariant and may be expressed in terms of either $F$ or $V_E$. We equate the two expressions for $q/a$. This relates the dependent variables, of course, but the connection is incomplete. The ray-tube area is singular at the aircraft, is represented by the variable $r$ in the local coordinates and by $A$ in the ground-fixed coordinates. We must then also relate the two

corresponding ray-tube area measures to a measure which is clearly galilean invariant. Such a galilean invariant measure is time $(t - t_o)$ elapsed from emission of a signal from the aircraft axis. In the local coordinates of the previous section, the quantity $r$ is related to this time by

$$r = a_o \cos \mu \, (t - t_o)$$

The definition (33) of the function $F$ leads to

$$\frac{q}{a} = \frac{F}{[a_o \cos \mu \, (t - t_o)]^{1/2}} \tag{42}$$

For the function $A$ we must carry out a local calculation of equation (26) near the aircraft. To lowest order we have

$$A = -\left[\left(1 + \frac{\sin \gamma}{\sin \mu \sin \theta_o}\right)\frac{\cos \theta_o}{\sin \theta_o}\frac{\partial \nu}{\partial \phi} + \frac{V \sin \phi \cos \mu \cos \gamma}{\cos \theta_o}\right.$$

$$\left. \times \frac{\cos^3\theta_o}{a_o^2 \sin^3\theta_o}\left(\frac{\partial c_o}{\partial \phi} - u_{t_o}\frac{\partial \nu}{\partial \phi}\right)\right](z_o - z)$$

The quantity $(z_o - z)$ is approximately equal to $a \sin \theta_o(t - t_o)$. The other terms may be re-expressed with the aid of equations (10), (27), and (29). After some calculation, we obtain

$$A = \frac{a_o \cos^2\mu \cos \theta_o \, (t - t_o)}{\sin \mu \sin \theta_o} \tag{43}$$

From equation (40) we obtain

$$\frac{q}{a} = \frac{(\sin \mu)^{1/2} \, V_E}{[\rho_o a_o^3 \cos^2\mu \cos^2\theta_o(t - t_o)]^{1/2}} \tag{44}$$

Equating the two expressions (42) and (44) for $q/a$ gives

$$V_E = \left(\frac{\rho_o a_o^2 \cos\mu \cos^2\theta_o}{\sin\mu}\right)^{1/2} F \tag{45}$$

This is the desired relation, giving $V_E$ in terms of the F-function.

Application in the program.- For each ray investigated, the function $V_E$ is computed from equation (45). This is expressed in terms of the independent variable $\xi$ obtained from equation (41).

Historical note.- The invariance result of Blokhintsev was found independently by Chernov in 1946 (ref. 23), but only in the special case of irrotational flow. Garrett (ref. 24) noted that Blokhintsev's result was equivalent to conservation of volume integrals of $\rho q^2/\Omega$ where $\rho q^2$ is the Rayleigh acoustic energy and $\Omega$ a frequency, both measured by an observer moving with the fluid. Hayes (ref. 25, 1968) showed that Garrett's result remains valid when the undisturbed flow field is unsteady, and that the quantity $\rho q^2 a A_n/\Omega^2 = \rho q^2 c_n \bar{c} \cdot \bar{A}/\omega^2 a$ is constant on a kinematic ray in this case, where $A_n$ is a ray-tube area cut by wave fronts and $\omega$ is a frequency measured by a fixed observer. Bretherton and Garrett (ref. 26, 1968) present a general theory governing wave motion in moving media.

## Signal Distortion and Age Variable

The purpose of this section is to define an "age" variable and to apply it in the calculation of the nonlinear distortion of the acoustic signal. This distortion appears in the propagation of acoustic signals over large distances. The distortion is caused by a weak, nonlinear effect resulting from small changes in propagation speed which are proportional to the strength of the signal. Although the nonlinear effect is locally weak, it is cumulative and, as a consequence, the total distortion of a sonic boom signal is far from negligibly small. Shock waves may appear in the signal or two shocks (or more) may merge into one. The process of distortion is governed by an age variable which is defined and applied in this section. The study of the location and motion of shock waves is to be covered in the following section.

We make here a simplifying assumption, one made by all investigators of sonic boom. We assume that in a lowest-order approximation the phase shift due to the change in propagation speed is the only nonlinear effect that needs to be considered. Thus, any nonlinear effect on the rays, the ray-tube areas, or the Blokhint-

sev invariance is assumed negligible. Although no comprehensive theory is available to fully justify this assumption, it is not made blindly. The neglected effects may be shown to correspond to higher-order terms in a small parameter for level flight in a uniform atmosphere (ref. 27, section on Finite Systems). In general, it appears likely that this assumption fails to be valid only when the assumptions of geometric acoustics fail or when the perturbations are no longer weak. Nonlinear effects on rays are discussed by Whitham in reference 16 and form an essential part of his method for predicting the trajectories of finite strength shocks (refs. 28 and 29).

The propagation velocity, equal to $\bar{c} = a\bar{n} + \bar{u}$ in the undisturbed fluid, is changed by $(\Delta a + q)\bar{n}$ , where $\Delta a$ is the perturbation in the speed of sound. This quantity is given by

$$\Delta a = \frac{\gamma_e - 1}{2} \frac{\Delta p}{\rho a} = \frac{\gamma_e - 1}{2} q$$

so that the change in propagation velocity is simply $(\gamma_e + 1)q\bar{n}/2$ .

If the phase were expressed as distance $s$ measured backwards normal to the wave fronts, the signal would experience a phase shift arising from the change in propagation velocity given by

$$\frac{ds}{dt} = - \frac{\gamma_e + 1}{2} q$$

where $t$ is time along the ray, that given by equation (18).

In treating the phase, we must distinguish between the actual phase variable and the phase variable according to linear theory. The nonlinear effect is basically the difference between the two. We term the actual phase $\xi_1$ , defined in the same way as before, and term the linear phase $\xi$ . The local distance phase $s$ is related with $\xi_1$ by $ds = c_n\, d\xi_1$ . The expression for the change in propagation velocity in terms of $\xi_1$ becomes

$$\frac{d\xi_1}{dt} = - \frac{\gamma_e + 1}{2} \frac{q}{c_n} = - \frac{\gamma_e + 1}{2} \frac{q}{c_o \cos \theta}$$

This equation describes the phase shift for a particular point on the signal, and thus with the linear phase $\xi$ fixed. The point on the signal found at $\xi$ according to the linear theory is actually found at $\xi_1$ , which is a function of $\xi$ and $t$ .

We express $q$ in terms of $V_E$ using equation (40), and obtain

$$\frac{d\xi_1}{dt} = - \frac{\gamma_e + 1}{2c_o} \frac{V_E}{\cos\theta\,(\rho A \sin\theta \cos\theta)^{1/2}}$$

The variable $t$ may be replaced by $-z$ through equation (18). We wish to transform the equation for the phase shift to a canonical form. We introduce the age variable $\tau$ defined by

$$\tau = \frac{\gamma_e + 1}{2c_o} \int_{-z} \frac{d(-z)}{a \sin\theta \cos\theta\,(\rho A \sin\theta \cos\theta)^{1/2}} \tag{46}$$

The phase shift is then governed by

$$\frac{d\xi_1}{d\tau} = - V_E(\xi_1, \tau) \tag{47}$$

in the canonical form desired.

With our basic assumption that the linear results for the ray-tube area and Blokhintsev invariance still hold with only the phase shifted, we must have

$$V_E(\xi_1, \tau) = V_E(\xi) \tag{48}$$

where $V_E(\xi)$ is the linear solution (independent of $t$ or $\tau$ because of the invariance). Here the actual phase $\xi_1$ satisfies equation (47) at constant $\xi$ and also the initial condition $\xi_1 = \xi$ at $\tau = 0$. The solution of equation (47) is then

$$\xi_1 = \xi - \tau V_E(\xi) \tag{49}$$

The linear phase $\xi$ is a function of $\xi_1$ and $\tau$ which may become multivalued in $\xi_1$. In this case the solution (48) and (49) must be modified to take into account the presence of shock waves. This modification is explained in the following section.

The solution (48) and (49) is the general solution to the first-order partial differential equation

$$\frac{\partial V_E}{\partial \tau} = V_E \frac{\partial V_E}{\partial \xi_1} \tag{50}$$

This equation may be considered a canonical one for inviscid wave propagation in one direction. One approach to the problem of calculating the nonlinear distortion lies in first deriving an equation of the form of equation (50). Equation (47) or its integral (49) gives the characteristics for equation (50) and its solution by standard methods is that of equation (48).

If $V_E$ is set equal to the $\xi_1$ derivative of a function $\mathcal{S}$ in equation (50) and the resulting equation is integrated with respect to $\xi_1$, the result is

$$\frac{\partial \mathcal{S}}{\partial \tau} = \frac{1}{2}\left(\frac{\partial \mathcal{S}}{\partial \xi}\right)^2 \tag{51}$$

We have here assumed that the function $\mathcal{S}$ is zero for sufficiently large negative $\xi_1$ with the consequence that the additive arbitrary function of $\tau$ from the integration is identically zero. In the following section, we define the function $\mathcal{S}$ as an integral over $\xi$ and use its properties to locate shocks. Equation (51) may be considered as an equivalent of equation (50). Actually, because of its properties when there are shocks, the function $\mathcal{S}$ contains more information than does $V_E$.

If the F-functions are obtained experimentally from wind-tunnel or flight test results at distances from the aircraft at which $\tau$ would not be negligible, a correction may be needed. This could involve a correction transforming F to an equivalent F as mentioned in the section, Flow Near the Aircraft. An alternative method would require calculating an initial value of $\tau$ in equation (46) which is not equal to zero.

If $\tau$ gets very large, the signal shape approaches that of an N-wave. With the shocks properly taken into account, the asymptotic behavior includes a maximum to $|V_E|$ which is proportional to $\tau^{-1/2}$ and a total phase difference between head and tail shocks which is proportional to $\tau^{1/2}$. The presence of the factor $\rho^{-1/2}$ in the integral of equation (46) indicates that for downward propagating rays in a real atmosphere the integral will be convergent; thus $\tau$ approaches a limiting value $\tau_{lim}$ as $z$ approaches $-\infty$. The signal in terms of $V_E$ approaches the limiting shape $V_E(\xi_1, \tau_{lim})$. Thus the signal shape "freezes" and does not become the ever-thickening N-wave of common farfield theory. This freezing effect is discussed in reference 1.

To illustrate the freezing effect a simple example is instructive. With level flight in a constant temperature atmosphere the asymptotic value of the age in a downward $(-\frac{1}{2}\pi > \phi > \frac{1}{2}\pi)$ direction is proportional to

$$\int_{-z_o}^{\infty} (z_o - z)^{-1/2} \exp[-\tfrac{1}{2}\beta(z_o - z)]\, d(-z) = \left(\frac{2\pi}{\beta}\right)^{1/2}$$

where $\beta^{-1}$ is the scale height of the atmosphere.

The corresponding finite integral in a homogeneous atmosphere is

$$\int_{-z_o}^{z} (z_o - z)^{-1/2}\, d(-z) = 2(z_o - z)^{1/2}$$

and the two integrals are equal if $z_o - z = \pi/2\beta$. The asymptotic signal shape in the real atmosphere is, therefore, the same as it would be in a homogeneous atmosphere at an altitude $\pi/2$ scale heights below the aircraft.

Application in the program.- For each ray, the variable $\tau$ is calculated from equation (46) at the same time the ray and ray-tube areas are calculated. At the ground the original phase $\xi$ is transformed to the actual phase $\xi_1$ by equation (49), giving thereby the function $V_E(\xi_1,\tau)$ at the ground. No provision is made for the correction required if the F-function is obtained from a measurement made a large distance from the aircraft flight axis.

Historical note.- The solution (48) and (49) was obtained by Poisson in 1807 (ref. 30) for plane waves in a constant-temperature gas ($\gamma_e = 1$). He obtained also equation (51) corresponding to this solution, with $\mathscr{S}$ a velocity potential. In 1860, Earnshaw (ref. 31) showed that with a polytropic gas the factor $\frac{1}{2}(\gamma_e + 1)$ which appears in equation (46) must be included in the analysis. An equation of the form of equation (50), although equivalent to Poisson's integral (51), seems not to have appeared before the issuance of a report of Chandrasekhar in 1943 (ref. 32).

With planar waves in a uniform gas, the age variable is simply proportional to the distance coordinate. The simplest cases with variable ray-tube area are those with cylindrical, spherical, or conical symmetry. In these cases, for N-waves in a uniform atmosphere, Landau (ref. 33) obtained the correct laws. The solution with a general signal, corresponding to that of this and the following section, was obtained by Landau and Lifshitz in 1944 (ref. 34) with cylindrical and spherical symmetry and by Whitham in 1952 (ref. 17) with axial conical symmetry (flow about a body of revolution). The application to flows with general conical symmetry, with the azimuth $\phi$ as an independent parameter, was made by Hayes in 1954 (ref. 27, section on Finite Systems). The arbitrary dependence of the solution of $\phi$ here involves the

principle that nonlinear effects on the rays are negligible. Ray-tube areas not based upon solution symmetry appeared in the age variable used by Rao (ref. 35, 1956) for maneuvering flight in a uniform atmosphere with straight line rays. Here the assumption that nonlinear effects on rays are negligible is implicit.

Variable atmospheric properties appear in the age variable $\beta$ used by Whitham (ref. 36, 1953) with spherical symmetry in a study of weak shocks in stars. Arbitrary area and fluid properties were combined by Hayes in 1957 (ref. 37) without consideration of winds. Several incorrect age definitions appeared in the Russian literature for the case with steady winds. In 1962 Ryzhov and Shefter (ref. 22) used the correct one but applied only to N-waves. Hayes (ref. 38, 1963) presented an analysis with winds similar to that of this section but made an error in the definition of the age (a factor $a/c_n$ is required under the integral); this paper includes an algorithm for ray-tube area with winds. Guiraud (ref. 19, 1965) uses an age variable, but one defined somewhat differently because he does not use Blokhintsev invariance or ray-tube areas.

The freezing effect, although inherent in the age variables appropriate with varying atmospheric density, appears not to have been discussed before reference 1. The integral involved with level flight in a constant temperature atmosphere is an error function and appears in a related form in the appendix to a 1955 paper of Busemann (ref. 39).

## Shock Location

The purpose of this section is to show how shock waves which appear in the distorted signal may be located. In the algorithm this is done at the same time the distorted signal is calculated. In general, with the age $\tau$ sufficiently large, the distorted signal $V_E(\xi_1,\tau)$ is multivalued in $\xi_1$ and, thus, physically meaningless. The actual signal contains one or more shock waves, and a proper treatment of the shocks eliminates the multivaluedness. A shock wave moves faster than the acoustic propagation speed in front of it and slower than the propagation speed behind it. Thereby, parts of the signal are propagated into the shocks (are "eaten up" by the shocks) and this phenomenon permits the remaining part of the signal to be single-valued.

The procedure for locating the shocks utilizes the function $\mathscr{S}(\xi_1,\tau)$ introduced in the preceding section. We drop the $\tau$ in the notation and write simply $\mathscr{S}(\xi_1)$ . In order to have $\mathscr{S}$ defined over the entire range of $\xi$ , it is defined as an integral over $\xi$ , or, equivalently, as a Stieltjes integral over $\xi_1(\xi)$ . Thus we write

$$\mathcal{S}(\xi_1) = \int_{-\infty} V_E(\xi)d\xi_1(\xi) = \int_{-\infty} V_E\left(\frac{d\xi_1}{d\xi}\right)d\xi \tag{52}$$

This function satisfies equation (51) over the entire range of $\xi$. The lower limit $-\infty$ simply means sufficiently negative to begin before the signal commences. The portions of the original signal $V_E(\xi)$ which appear in the actual signal correspond to segments or branches of the multivalued function $\mathcal{S}(\xi_1)$, chosen in such a way as to make up a single-valued function.

The bisector law for a weak shock states that it moves with a normal speed midway between the acoustic propagation speed in front of and behind the shock. This law may readily be proved, and requires only that the curvature of the fluid isentrope be continuous. In terms of our notation, the law states that

$$\left(\frac{d\xi_1}{d\tau}\right)_{\text{shock}} = -\frac{1}{2}(V_{E_+} + V_{E_-}) \tag{53}$$

where $+$ and $-$ indicate the two sides of the shock. We use [ ] to denote jump in a quantity; thus $[\mathcal{S}] = \mathcal{S}_+ - \mathcal{S}_-$. The $\tau$ derivative along the shock of $[\mathcal{S}]$ is calculated using equations (51) and (53), and is

$$\frac{d[\mathcal{S}]}{d\tau} = \left[\frac{d\mathcal{S}}{d\tau}\right] = \left[\frac{\partial\mathcal{S}}{\partial\tau}\right] + \left(\frac{d\xi_1}{d\tau}\right)_{\text{shock}}\left[\frac{\partial\mathcal{S}}{\partial\xi_1}\right]$$

$$= \frac{1}{2}(V_{E_+}^2 - V_{E_-}^2) + \left(\frac{d\xi_1}{d\tau}\right)_{\text{shock}}(V_{E_+} - V_{E_-}) = 0$$

The function $\mathcal{S}$ is continuous in the original signal at $\tau = 0$, so that $[\mathcal{S}]$ is initially zero at the point of formation of any shock. Thus, $\mathcal{S}$ remains continuous across any shock. It is clear that this property is preserved when two shocks merge into one, and that the property thus holds in general.

We conclude then that the single-valued function of $\xi_1$ made up of segments of $\mathcal{S}(\xi_1)$ is a continuous function. This result we may term the equal-area law. In its simplest form the law states that a shock, which appears when the final curve jumps from one segment of $\mathcal{S}(\xi_1)$ to another, is located where it cuts off equal

areas from the multivalued curve $V_E(\xi_1)$ . Figure 11 shows in a simple case the curves for $V_E(\xi)$ , $V_E(\xi_1,\tau)$ , $\mathscr{S}(\xi_1)$ , and $\Delta p(\xi_1,\tau)$ .

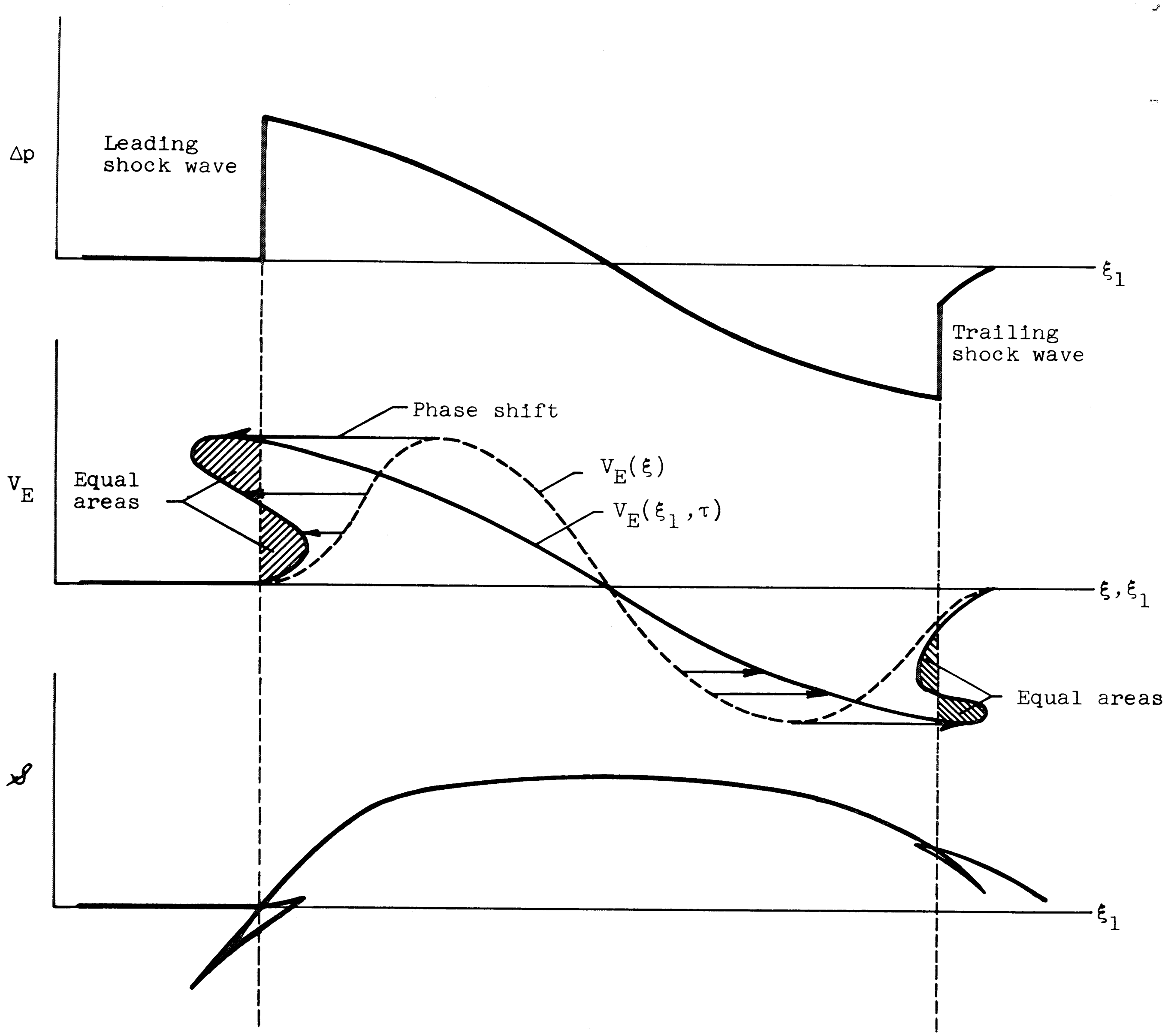



Figure 11. Distortion of signal

Two important properties of $\mathscr{S}(\xi_1)$ are not evident from the analysis above, and we state them without proof. One property is that there is only one continuous function made out of segments of $\mathscr{S}(\xi_1)$ for which $d\xi_1/d\xi > 0$. The principal consequence of this property is that we do not have to trace the trajectories of the shocks from $\tau = 0$ but may locate them at a fixed value of $\tau$ without considering their history. The second property is that the desired function is precisely $\sup \mathscr{S}(\xi_1)$, the superior limit of $\mathscr{S}(\xi_1)$, defined to be the function obtained by taking the largest value of $\mathscr{S}$ available for each value of $\xi_1$. This property simplifies significantly the process of locating the shocks in a complicated case.

The equal-area law may be established in other ways than that of using the bisector law. One alternative approach invokes the principle of conservation of mass, identifying $\mathscr{S}$ with a Lagrangian variable or with a particle displacement variable. Another approach involves establishing equations with viscosity and using the limiting process $\mu \to 0$. In the second of these alternative approaches, the property that the desired function is $\sup \mathscr{S}(\xi_1)$ appears naturally.

The process of locating the shocks may now be described. The function $\mathscr{S}(\xi_1)$ is obtained, and the single-valued function $\sup \mathscr{S}(\xi_1)$ is identified. The function $V_E(\xi_1, \tau)$ is then plotted, retaining only those segments of $\xi_1$ which appear in $\sup \mathscr{S}$. The corresponding function $\Delta p(\xi_1, \tau)$ is calculated and gives the desired pressure signature. The parts of the original signal corresponding to segments of $\xi_1$ not represented in $\sup \mathscr{S}$ are the parts eaten up by the shocks. Where there is a jump from one segment of $\xi_1$ to another in $\sup \mathscr{S}$, there is a corresponding jump in $V_E$ and $\Delta p$, and such a jump represents the shock.

The function $\mathscr{S}(\xi_1)$ may be obtained by direct integration, but a somewhat different construction is advantageous. Together with the original function $V_E(\xi)$, we require its integral

$$S_0(\xi) = \int_{-\infty} V_E \, d\xi \tag{54}$$

From equation (49) we have

$$\frac{d\xi_1}{d\xi} = 1 - \tau \frac{dV_E(\xi)}{d\xi}$$

at constant $\tau$. Applying these relations to the integral (52), we obtain

$$\mathscr{S}(\xi_1) = S_o(\xi) - \frac{1}{2}\tau\, V_E(\xi)^2 \tag{55}$$

with equation (49) giving the relation between $\xi_1$ and $\xi$. Equations (49) and (55) may be combined to give the relation

$$\mathscr{S}(\xi_1) = S_o(\xi) - \frac{(\xi_1 - \xi)^2}{2\tau} \tag{56}$$

The graphical method of Burgers (ref. 40) is a method of finding $\sup \mathscr{S}(\xi_1)$ from a given "summation" curve $S_o$, using equation (56).

The pressure signature $\Delta p$ is to be calculated from $V_E$ using equation (40) and the relation $\Delta p = \rho a q$ (equation (39)). We are usually interested in the pressure at the ground, and must take into account the reflection of the wave system from the ground. This reflection may be visualized as coming from the mirror image with respect to the ground of the impinging wave system, and the reflected pressure signature appears superimposed on the incoming one. Very near the ground, say at a height $h$, the reflected pressure is the same as the incident pressure, with a phase delay (in $\xi_1$) equal to $2h/a \sin\theta$. If we assume that $h$ is small enough that the phase delay may be neglected, the effect of the ground is simply to double the pressure. This factor appears as a reflection factor equal to 2.

In practice, an empirical reflection factor $K_R$ other than 2 is often used. The final expression for the pressure signature, from equation (40), is then

$$\Delta p(\xi_1) = K_R\, V_E(\xi_1, \tau)(\rho a^2/A \sin\theta \cos\theta)^{1/2} \tag{57}$$

The pressure signature far from the ground, with no reflection taken into account, is obtained from equation (57) simply by setting the factor $K_R$ equal to 1. Our analysis and algorithm for calculating sonic boom signatures are completed by equation (57). The ray tracing and ground intersection calculations discussed earlier tell us where and when the pressure signatures occur.

The pressure on the ground at any instant is approximately constant along the ground intersections of the wave fronts ( the $y_1$ direction), and has its main variations in the direction normal to these ground intersections (the $x_1$ direction). Distance in a plane $z$ = const. normal to the wave front intersections (in the $x_1$ direction) from the zero phase wave front may be used as a phase for describing the pressure on that plane at a given instant. This distance phase variable is equal to $c_o\xi_1$. This phase is galilean invariant; it was not used in the analysis as the basic variable because $\xi_1$ appeared to be a more useful quantity.

Application in the program.- For each ray, the function $S_0(\xi)$ is obtained from equation (54), and the multivalued function $\mathcal{S}(\xi_1)$ from equations (55) and (49). The branches corresponding to sup $\mathcal{S}(\xi_1)$ are identified, and $\Delta p(\xi_1)$ is calculated from equation (57).

Historical note.- Shock waves remained ill understood through almost all of the nineteenth century. The bisector law was given by Crussard in 1913 (ref. 41), who described what we would term a half N-wave in a constant-area duct. This law was used by Landau and Lifshitz (ref. 34, 1944) to obtain the equal-area law (see also ref. 17). In 1950, E. Hopf (ref. 42) gave a thorough study of Burgers equation with particular attention to the limiting process $\mu \rightarrow 0$. This study establishes the equal-area law through this limiting process and serves as the basis for Burgers' graphical method (ref. 40). Landau (ref. 43, 1945) mentioned conservation of mass as the basis for the equal-area law, but without giving details. Lighthill (ref. 44, 1956) noted that the equal-area law is equivalent to continuity of a Lagrangian variable. Middleton and Carlson (ref. 45, 1965) applied the equal-area law in practical calculations, obtaining $\mathcal{S}(\xi_1)$ through direct integrations.

## A Note on Viscous Effects

Although they are not properly part of this analysis, this exposition would be incomplete without some mention of viscous effects. In sonic boom problems of practical interest, viscous effects are sufficiently weak so that their only effect is to give finite thickness to shock waves. This thickness, though finite, is generally several orders of magnitude smaller than any of the other characteristic scales of the problem. Hence the treatment of shocks as strict discontinuities is completely sound. In certain other problems, as for example of acoustic propagation at ionospheric heights, viscous effects are important. Here we indicate briefly the governing equation, and describe a few of its properties.

As in much of our development, we take advantage here of the fact that the wave system is approximately a plane wave system. The viscous effects are considered to comprise only viscosity and heat conduction. They appear, in a coordinate system (s,t) moving with the undisturbed inviscid propagation speed, in a diffusion term of the form

$$\frac{\partial V_E}{\partial t} = \frac{1}{2\rho}\left(\frac{4}{3}\mu + \mu' + \frac{(\gamma_e - 1)k}{c_p}\right)\frac{\partial^2 V_E}{\partial s^2} \qquad (58)$$

Here $s$ is the distance coordinate which serves as the phase variable, $\mu$ and $\mu'$ are the shear and dilatational viscosity coefficients, $\gamma_e$ is the ratio of specific heats of the gas, $c_p$ is the specific heat capacity at constant pressure, and $k$ is the heat conduction coefficient. The variable $V_E$ in equation (58) may be any intensity measure, but it is identified here as the $V_E$ defined earlier. The nonlinear term corresponding to that in equation (50) has been omitted.

We next change the variables $t$ and $s$ to the corresponding variables $\tau$ and $\xi_1$ and include the nonlinear term. The result is

$$\frac{\partial V_E}{\partial \tau} = V_E \frac{\partial V_E}{\partial \xi_1} + \frac{1}{2} N(\tau) \frac{\partial^2 V_E}{\partial \xi_1^2} \tag{59}$$

where

$$N = \frac{1}{\rho}\left(\frac{4}{3}\mu + \mu' + \frac{(\gamma_e - 1)k}{c_p}\right) \frac{dt}{d\tau}\left(\frac{d\xi_1}{ds}\right)^2$$

is a positive reduced kinematic viscosity. The quantity $ds/d\xi_1$ is simply $c_n = c_o \cos\theta$, while

$$\frac{dt}{d\tau} = \frac{2c_o \cos\theta(\rho A \sin\theta \cos\theta)^{1/2}}{(\gamma_e + 1)}$$

from equation (46). We obtain thereby

$$N = \frac{2(\rho A \sin\theta \cos\theta)^{1/2}}{(\gamma_e + 1)\rho c_o \cos\theta} \left(\frac{4}{3}\mu + \mu' + \frac{(\gamma_e - 1)k}{c_p}\right) \tag{60}$$

Equation (59) is a generalized Burgers equation, generalized in that the quantity $N$ may be a general function of $\tau$.

Lighthill (ref. 44) introduced the concept of the Reynolds number of a lobe of a solution of Burgers' equation. As applied to equation (59), we identify a lobe as that part of the signal

between two consecutive zeros of $V_E$, at $\xi_1 = a$ and $\xi_1 = b$. Lobes can disappear with increasing $\tau$, but no new lobes can appear. The first moment $\mathfrak{M}_1$ of the lobe is

$$\mathfrak{M}_1(\tau) = \int_a^b V_E \, d\xi_1$$

The Reynolds number of the lobe is

$$\operatorname{Re}(\tau) = \frac{2|\mathfrak{M}_1|}{N} \tag{61}$$

If the Reynolds numbers of all the main lobes of a signal are very large, viscous effects are unimportant and only determine shock thicknesses. If the Reynolds number of the largest lobe is small, nonlinear effects are unimportant.

The first moment of a lobe obeys the equation

$$\frac{d\mathfrak{M}_1}{d\tau} = \tfrac{1}{2}N\left[\left(\frac{\partial V_E}{\partial \xi_1}\right)_b - \left(\frac{\partial V_E}{\partial \xi_1}\right)_a\right] \tag{62}$$

From this result it may be shown that $|\mathfrak{M}_1|$ cannot increase. It is constant for $N = 0$ only if there are no shocks in the lobe for which $V_E$ changes sign. The Reynolds number can increase, but only if $N$ decreases sufficiently fast with $\tau$.

The second moment of a lobe is defined

$$\mathfrak{M}_2 = \int_a^b V_E^2 \, d\xi_1$$

and represents the wave energy in the lobe. It satisfies the equation

$$\frac{d\mathfrak{M}_2}{d\tau} = - N \int_a^b \left(\frac{\partial V_E}{\partial \xi_1}\right) d\xi_1 \tag{63}$$

This moment cannot increase. It is constant for $N = 0$ only if there are no shocks in the lobe. These results were given in the paper for which reference 37 is the abstract.

Application in the program.- Viscous effects are not included in this program.

Historical note.- The combination of coefficients which appears in equation (58) and which governs viscous weak wave propagation appeared in a 1910 paper of G.I. Taylor (ref. 46) on the structure of weak shock waves (with $\mu' = 0$). Lighthill (ref. 44, 1956) showed that Burgers' equation was the appropriate one for general planar weak wave propagation for a calorically perfect gas. A generalized Burgers equation of the form (59) was obtained by Hayes (ref. 37, 1957) without winds. Hayes (ref. 47, 1958) also showed that the combination of coefficients in equation (58) was appropriate with a general equation of state as well as for a perfect gas. The inclusion of winds in a derivation of equation (59) was done by Hayes (ref. 38, 1963) but with an error coming from the error made in the definition of $\tau$. An equation of the form (59) was obtained by Guiraud (ref. 19, 1965) in terms of his equivalent of the variable $V_E$. Further details and discussion of viscous effects may be found in references 44 and 47.

## Summarizing Statement

At this point, the analysis is complete and ready to be used for computing sonic boom. Here we briefly summarize the algorithm for obtaining pressure signatures in the form realized in the computer program.

To compute sonic boom pressure signatures in a stratified atmosphere:

1. Specify input data.

    a. The thermodynamic properties of the atmosphere and the horizontal wind velocities are needed as functions of the altitude $z$ and are used throughout the calculation.

    b. The aircraft Mach number $M$, its heading angle $\psi$, its climb angle $\gamma$, and its bank angle $\phi_a$ are needed as functions of the time $t_a$. The initial location of the aircraft is needed to start the trajectory calculation. As an option, aircraft load factors may be specified as functions of time $t_a$.

    c. F-functions for the aircraft are needed in sufficient number to serve over the range of aircraft Mach number covered in the trajectory.

2. Calculate aircraft maneuver functions using 1b.

   a. The aircraft trajectory, $x_a(t_a)$ , $y_a(t_a)$, and $z_a(t_a)$, is calculated.

   b. The derivatives of heading, climb, and Mach angles, $d\psi/dt_a$, $d\gamma/dt_a$ , and $d\mu/dt_a$ , are calculated, either directly from the input data or (optionally) from aircraft load factors. These derivatives are needed in the ray-tube area calculation 4b.

3. Calculate the initial wave normals using 1b.

   The orientations of the initial wave normals, parametrized by their azimuth angle $\phi$ , are needed to determine the invariants $c_o$ and $\nu$ used in the ray calculations 4.

4. Calculate rays and functions along rays using 3.

   a. The rays, described by functions $x(t_a,\phi,z)$, $y(t_a,\phi,z)$, and $t(t_a,\phi,z)$, determine where and when the sonic boom signal hits the ground.

   b. The ray-tube area $A$ is calculated along each ray using 2b. This ray-tube area is used in calculating both the age 4c and the final pressure signature 6c.

   c. The age $\tau$ is calculated along each ray. This is used in calculating the signal distortion 6.

5. Calculate the linear acoustic signal on each ray.

   The function $V_E(\xi)$ is obtained from an F-function by a transformation of both dependent and independent variables. The integral $S_o(\xi)$ of $V_E$ is also calculated.

6. Calculate the distorted pressure signature.

   a. For the age at the ground, calculate the distorted signal $V_E(\xi_1,\tau)$ and its integral $\mathscr{S}$ .

   b. Locate the shock waves using $\mathscr{S}$ .

   c. Calculate the final pressure signature $\Delta p(\xi_1)$ using a selected reflection factor. This is the desired output.

The logical order of the algorithm follows closely that of the theoretical analysis, with one exception. The age is computed at the same times as are the rays and ray-tube area rather than later with the distorted signal.

## COMPUTER PROGRAM

### General Description

The computer program for solving the preceding sonic boom equations has been written in FORTRAN with the aim of obtaining a versatile computational technique adaptable to a variety of computers. This program was developed on an IBM-1130 and has also been run on a CDC-6600. A complete program listing for the CDC-6600 is given in Appendix A, and a sample printout appears in Appendix B. This printout will be discussed in detail in the next chapter, COMPUTATION RESULTS.

The organization of the program is shown in figure 12. The input processor reads input data and converts them to appropriate units and format for use in the subroutines. The input data are then listed on the output sheets (Appendix B) for identification and checking purposes. The program next computes and lists out pertinent maneuver data at the selected initial time $t_a$ . These outputs include aircraft location, Mach number, direction angles, and derivatives. The program then proceeds to the ray path and ray-tube area calculations beginning with the ray parameters $\phi = 0$ and $t_a$ . The program computes the ray trajectory, angle with the horizontal $(\cos \theta)$ of the wave normal, the ray-tube area, and the age variable. These outputs are listed as functions of altitude $z$ ; the time and $(x,y)$ coordinates are cumulative from the first maneuver point. When a stopping condition for the ray trajectory is reached, such as $z = 0$ , the program computes phase variables and the $\mathcal{S}$-function, and utilizes the F-function to determine the distorted pressure signature. These output quantities are then listed in the order of increasing values of L/LA . At each ray intersection with the ground, the program stores parameters which give the location, time, and maximum pressure of the calculated signal. The program then returns to the same maneuver point $(t_a)$, increments the ray azimuth angle $\phi$ , and proceeds again through the ray tracing and pressure calculations. When calculations for the set of rays are completed corresponding to the maneuver point $t_a$ , the program proceeds to the next maneuver point $t_{a+1}$ and the ray tracings are repeated. In the event the ray-tube area diminishes to zero during its traverse, or if the ray becomes horizontal, the calculation along that ray stops. After completing all of the maneuver, ray tracing, and pressure sequences, the summary data which have been saved for each ray are used. Linear interpolations are made to determine the locations of the rays and maximum pressures at the same elapsed time $t$ . Thus, ground shock intersection coordinates are made available, together with indicators of boom intensity there.

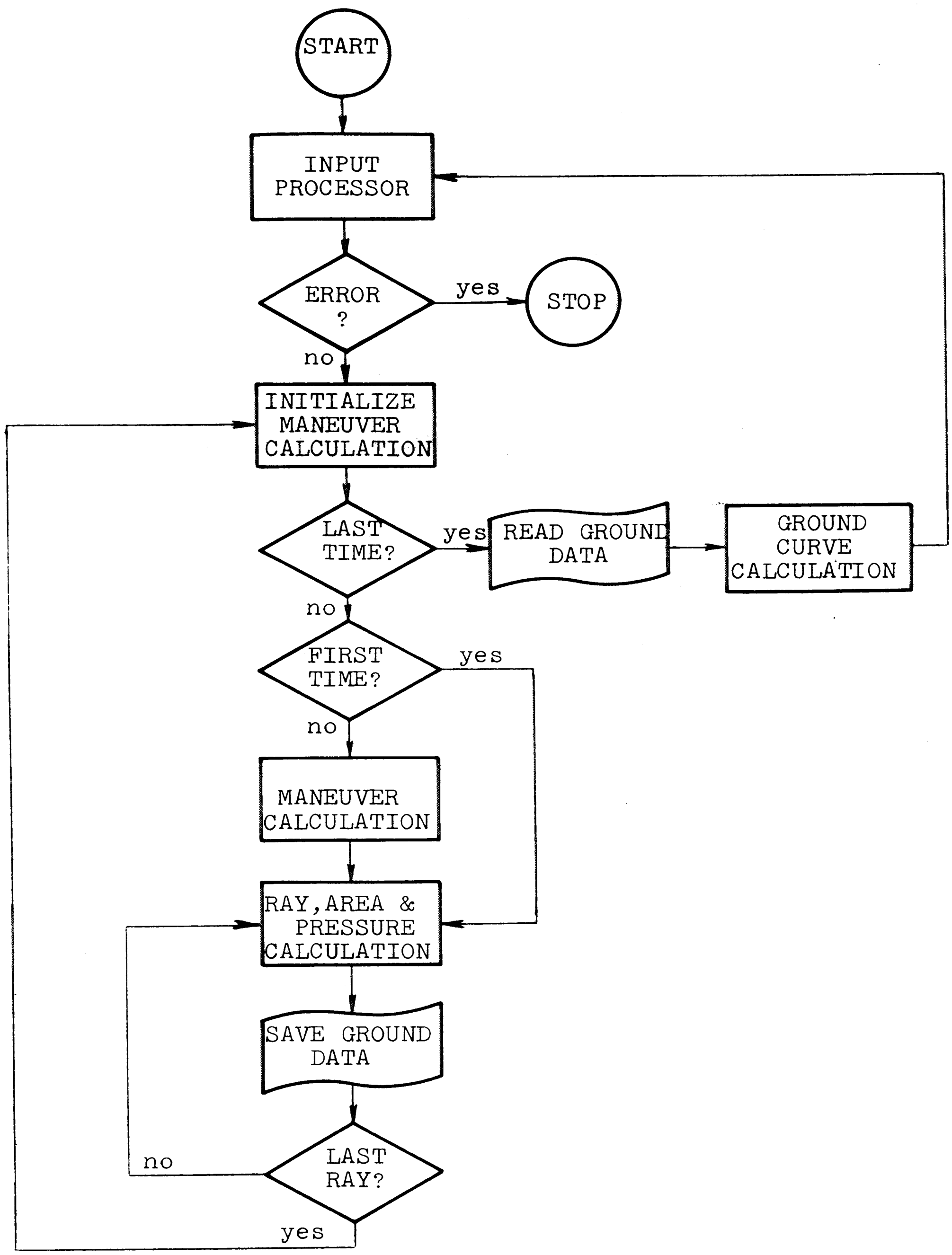



Figure 12. General flow chart

## Subroutines and Operations

The program consists of a master program, SONIC, and nineteen subroutines. Figure 13 gives a representation of the relationships among these subroutines. A brief description of their functions is given in the following paragraphs. Numbers correspond to those shown on the flow diagram.

1. SONIC This is the master program which initializes and calls the subroutines. It also serves as the main program for the maneuver calculation.

2. INPU This subroutine reads in all input data, checks them for self-consistency and validity, and prints out a copy of the data for comparison and problem identification.

3. ATMO This program calculates atmospheric data when using the standard atmosphere option. For a given altitude $z$, the corresponding temperature, pressure, density, and speed of sound are found. The altitude above sea level must be less than 170 600 ft (52 km).

4. ANGLE This routine corrects input data so that no two consecutive angles are more than $180^\circ$ apart; for example, $25^\circ$ and $-190^\circ$ would be changed to $25^\circ$ and $170^\circ$, respectively.

5. DERM2 This routine calculates the integrands used in solving for the maneuver path.

6. OUTM2 This program controls printing of the maneuver output. When a printout point is reached, subsidiary variables such as $q_\infty$ and $\mu$ are calculated.

7. INPOL This is a routine to perform linear interpolation for a single variable.

8. LAGRA This program carries out quadratic interpolation for nonequispaced points. The function value and/or its first derivative is produced.

9. AREA This program controls the calculation of the rays, area, and pressure. It saves the ground intersection points and provides for the calculation of all $\phi$'s .

10. DERIZ The calculation of the integrands for the ray and area integrals is carried out by this routine.

11. OUTPU This program calculates the age function using Simpson's rule for integration. It provides for printing ray, area, and age data at printout points.

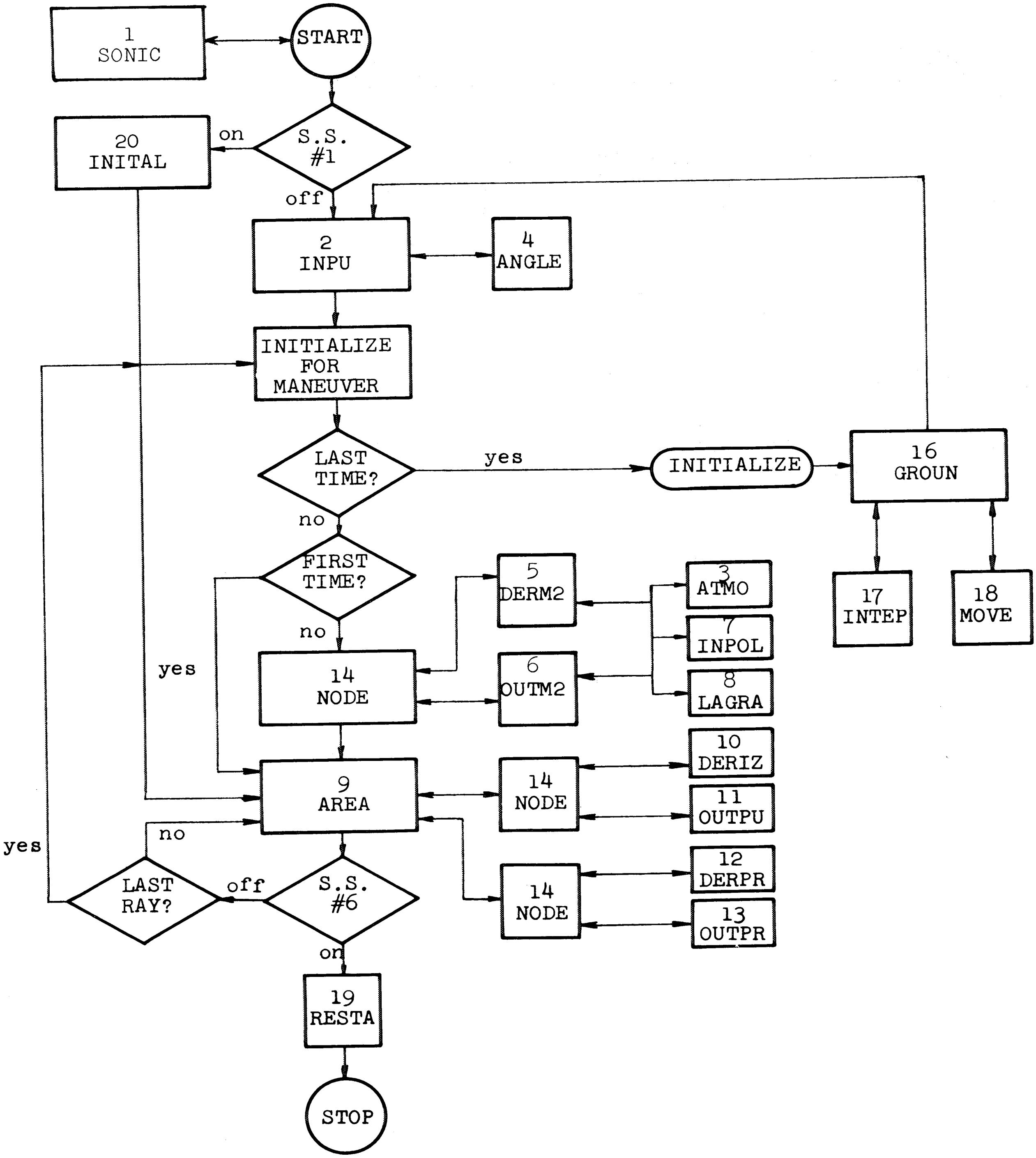



Figure 13. Program organization

12. DERPR  This program calculates the integrand required by the pressure calculation.

13. OUTPR  Evaluation of phase distortions $\xi$ , $\delta$-functions, and pressure data, and printing of them, are carried out by this routine.

14. NODE  This routine provides for the solution of a set of n first-order ordinary differential equations or integrals. A Runge-Kutta-Gill (RKG) starting procedure and a highly stable Adams-type predictor-corrector method are used.

15. PASS  This is an auxiliary routine used by NODE when evaluating integrals.

16. GROUN  This program retrieves the ground intersection data and reshuffles it to provide tables of ray location and maximum pressure in order of increasing time for each maneuver point. It then determines the ground intersection contours for equal time on the ground.

17. INTEP  This program provides for the simultaneous linear interpolation of x , y , and pressure for a given value of time.

18. MOVE  This routine moves tabular data from location J to location K .

19. RESTA  This is a routine used to save data for restarting the problem at a later time.

20. INITAL  This routine provides for starting a problem from previously saved data.

Three sense switches (SS) may be used as follows:

| Sense Switch | Function |
|---|---|
| SS-1 | Restart solution from data saved on Tape 9. |
| SS-5 | Print out during all integrations if trouble is suspected. |
| SS-6 | Stop calculations at end of next ray trace and pressure calculation. Save data on Tape 9 for future restart. |

The names of variables (FORTRAN symbols) which are used in the program are identified in Table 1. Comments which will be listed by the program when appropriate are shown in Table 2, including suggestions for correcting errors.

## Program Utilization and Instructions

This section describes how to set-up the inputs for utilization of the digital program and explains applications of the equations presented in THEORETICAL ANALYSIS. Details which are needed for setting up a card input deck are presented in Tables 3, 4, and 5 and explained in the following paragraphs. Table 3 shows how input data are arranged on punched cards, giving the data formats and notes relating to their set up. Table 4 presents definitions of the input nomenclature and option numbers, whereas Table 5 lists the units used in the three choices of input units. The following subsections are labeled to correspond to related sections of THEORETICAL ANALYSIS.

The Atmosphere.- The inputs of tabular atmospheric data are made on the sets of cards labeled Cards 5, 6, and 7 in Table 3. The temperature and either the pressure or the density are tabulated for various arbitrary altitudes in ascending order. Either the pressure or the density table is used, not both. A separate card is used for each altitude. The units are selected according to Table 5, choosing a number for IUNIT according to Table 4. If the 1962 U.S. standard atmosphere is desired, ISTAN = 1, and Cards 5 and 6 are omitted from the deck. The wind magnitude and wind direction are listed for various altitudes on Cards 7. The altitude on Cards 5, 6, and 7 may be geopotential or geometric, selecting a corresponding digit for IALT (Table 4).

These temperature, pressure, and density tables are interpolated linearly, whereas the wind table is interpolated quadratically. The number of altitudes is limited to 25 and should be selected carefully depending on their change in gradients to avoid unacceptable errors in the interpolation.

Aircraft Maneuvers. - Physical data regarding the aircraft are placed on Card 3. The altitude HG is the height of the ground above sea level whereas HO is the initial altitude of the aircraft above sea level. The initial values of the x,y coordinates of the aircraft are set to zero within the program. Its position and altitude at later times are calculated using equations (2) based on the maneuver inputs of Cards 8. The wing loading WS is used only in an auxiliary calculation of force coefficients $C_L$ and $C_T - C_D$ (eqs. 3 and 4), and does not enter into the sonic boom calculation here (except indirectly through the F-function values). The aircraft length LA is used as a reference length, whereas the radius of the earth is used in conversions between geopotential and geometric altitudes in the atmospheric functions.

The maneuver information is placed on Cards 8 as functions of increasing flight time. For Option 1 (IM = 1) the axial and

lifting load factors, Mach number, flight path angle, heading angle, and bank angle are input using a separate card for each flight time. The inputs should, of course, be made consistent; that is, the Mach number and angles should be values which correspond to the load factors $n_T$ and $n_L$ . Equations (5) and (7) are used to obtain the time derivatives. For Option 2 (IM = 2), the load factors are not required. These tables are interpolated quadratically, so that at least three maneuver cards are needed. The time derivatives are obtained by calculating slopes of quadratic curves fitting the data. On Card 9 are given the initial time and subsequent time intervals along the flight path for which rays are to be initiated.

The force coefficients are determined using equations (3) and (4) with the dynamic pressure $q_\infty$ calculated in the program. In Option 2, the load factors used in these equations are given by $n_L = \cos\gamma$ and $n_T = \sin\gamma$ to provide approximations to complete the digital program output listing. This substitution is exact only for nonaccelerating, straight flight.

Initial Wave Normals. - The ray parameters are specified by the time $t_a$ (Card 9) and the azimuth angle $\phi$ . This angle is first set to zero within the program, and then incremented by an amount $\Delta\phi$ (DELFI) after each ray-path integration until $\phi_{MAX}$ (FIMAX) is exceeded, as input on Card 10. The negative increments $-\Delta\phi$ are then taken until $\phi_{MAX}$ is exceeded. For a problem which is symmetric in $\phi$ , the input DELFI can be negative to avoid duplicate calculations of the pressure signatures at $+\phi$ and $-\phi$ . The values of $\nu$ and $\sin\theta_0$ are evaluated using equations (9) and (10).

Mach Conoids and Ground Intersections. - Each time a ray trajectory intersects the ground, its location, arrival time, and peak pressure are saved in a special table. This peak pressure is selected from the array of pressures ($\Delta p$ as functions of phase $\xi$) which are determined, but not saved in other parts of the program.[1] After all maneuver, ray tracing and pressure calculations have been completed, these saved data are interpolated to obtain intersection data at specific arrival times at the ground, as follows.

---

1) These pressure values may be useful indicators of the actual pressure increment at each location, but they are not accurate. The peak values have not yet been adjusted, at this point of the procedures, for the distortion of the signature and the development of shock-wave structure. The user must judge the value of these interpolated peak pressure listings and, for accurate values, properly process the signature data given in the detailed tables for each ray-tube calculation.

The program selects from the table the minimum arrival time $t$ for the set of rays which left the aircraft at the second maneuver time $(t_{a+1})$. Then the program proceeds to the previous ray set corresponding to $t_a$ and interpolates linearly to find the ray locations and peak pressures for the same arrival time $t$. There are generally two of these, corresponding to starboard and port ray azimuths. The program then finds a new minimum arrival time for the set of rays corresponding to $t_{a+2}$ and interpolates the preceding sets $(t_a$ and $t_{a+1})$. This process continues through each set of rays corresponding to subsequent aircraft maneuver times.

An example of output for this ground intersection table is shown in figure 14. These data were obtained for large ray azimuth increments of $20^{\circ}$ and, therefore, yield somewhat crude interpolations. The maneuver flight times at which ray calculations were initiated were at one-second increments. The solid-line curves show ground intersection locations for sets of rays for these maneuver times. The dashed-line curves show ground intersection interpolations for the fixed arrival times of 37.6, 38.4, 39.3, and 40.1 seconds.

Snell's Law and Ray Tracing. - The ray tracing initial integration step (ZSTEP) and print intervals (ZPRN) for the solution of equations (18) are specified on Card 10. Equation 16 is calculated for $c_o$. A quantity $c_{oo} = c_o + u_{no}$ is also obtained. The values of $\phi$, $\nu$, $c_{oo}$ and $\bar{c}_o$ are listed at the head of each ray tracing table (Appendix B). The wind components $u_n$ and $u_t$ are obtained using equations (15) and $\cos\theta$ is obtained using equation (17). The ray trajectory integration step is halved or doubled as the calculations proceed from the aircraft to the ground, according to predictor-corrector error limits and comparisons made in sub-routine NODE. Listings of ray, area and age variables are made at descending altitude increments specified by ZPRN. These listings are also given at the altitude corresponding to the next integration interval below the selected altitude called PRIN input on Card 10.

The calculations along a given ray are stopped at the ground $(z = 0)$, or if the ray-tube area converges to zero, or if the ray angle decreases to within $2^{\circ}$ of horizontal $(\cos\theta = 0.9994)$. The additional displacement during the final $2^{\circ}$ to the true horizontal point is approximately

$$\Delta\bar{r} = \frac{2^{\circ}}{57.3^{\circ}/\text{rad}} \frac{\bar{c}}{d(a - u_n)/d(-z)}$$

with the quantities evaluated at the horizontal point. In certain special problems, more than one $\Delta\bar{r}$ contribution may be

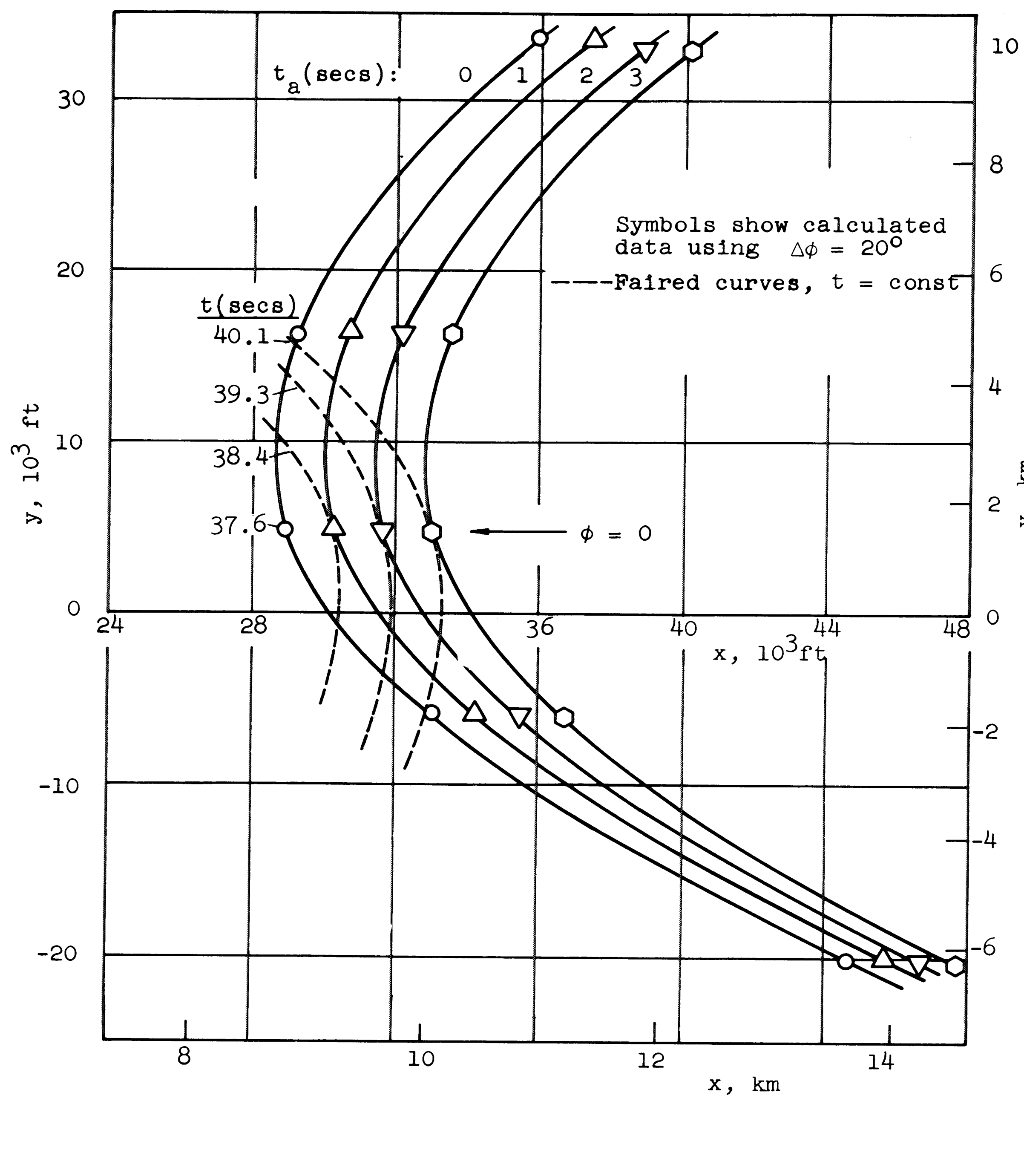



Figure 14. Typical ground intersection data

needed. For standard atmosphere at sea level without winds, the correction is

$$|\Delta\bar{r}| = 1.93 \text{ stat mi} \quad (3.1 \text{ km})$$

Ray-tube Area. - The partial derivatives of $c_o$ and $\nu$ with $t_a$ and $\phi$ are calculated in subroutine AREA using equations (27), (28) and (29). Variables dependent on altitude $z$ and the integrals (eqs. (24)) are evaluated using subroutines DERIZ and NODE, and the various terms of the area equation (26) are combined in subroutine OUTPU. The values of the ray-tube area are listed in the output table as a function of altitude $z$.

Flow Near the Aircraft. - The program input of the aircraft F-function is made of Cards 4. $F_i$ is tabulated as a function of length parameter L/LA. The input format permits specifying the $F_i$ vs. L/LA function for several azimuth angles $\phi_r$ (PHI) relative to the aircraft vertical (butt-line). The program selects that $F_i$ vs. L/LA curve given in the input table which has $\phi_r$ closest to the current computed value $\phi_r = \phi - \phi_a$, where $\phi$ is the ray parameter and $\phi_a$ is the bank angle of the aircraft. The factor $F_f$ is input on Card 10, and multiplies $F_i$ before it is listed in the output tables (eqs.(34)).

This format for inputing the aircraft's signal requires that the dependency of $F$ on $\phi_r$, $C_L$, weight and Mach number is determined independently of this sonic boom program. Other subroutines for calculating variations in $F_i$ may be substituted, however, by a programmer. Also, the dimension specifications in this program may need to be changed to allow for larger input tables.

This program calculates pressure signature parameters at each given value of L/LA and at three additional intermediate stations, using linear interpolation to determine the intermediate values of the F-function. These extra data are useful for determining the pressure curve and the $\mathcal{S}$-curve which yields the shock locations.

Geometric Acoustics and Blokhintsev Invariance.- The quantity $V_E$ (eq. (45)) is evaluated in subroutine DERPR. The phase $\xi$ (eq. (41)) is determined in OUTPR. These are listed in the output tables (Appendix B) for corresponding values of L/LA.

Signal Distortion and Age Variable.- Equation (46) for the age variable must be rewritten for the digital computations in order to properly handle its initial singularity. This -1/2 power singularity occurs because the initial ray-tube area is zero. Its evaluation is as follows:

The age variable is

$$\tau_a = \frac{1.2}{c_o} \int_{z_o}^{z_1} \frac{d(-z)}{\sqrt{G_5 G_4^3 \sqrt{A(z)}}} \tag{64}$$

where $z_o$ is the airplane altitude where the ray begins and $z_1$ is any lower altitude. $G_o$ through $G_{13}$ represent combinations of terms in the area equation (26) which are used in the program and defined in Table 1.

Let

$$Q(z) = \frac{1}{\sqrt{G_5 G_4^3}} \frac{\sqrt{z_o - z}}{\sqrt{A}} \tag{65}$$

so that

$$\tau_a = \frac{-1.2}{c_o} \left[ \int_{z_o}^{z_1} \frac{Q(z_o)\, dz}{\sqrt{z_o - z}} + \int_{z_o}^{z_1} \frac{Q(z) - Q(z_o)}{\sqrt{z_o - z}} dz \right]$$

$$= \frac{-1.2}{c_o} \left[ -2Q(z_o)\sqrt{z_o - z} + \int_{z_o}^{z_1} \frac{Q(z) - Q(z_o)}{\sqrt{z_o - z}} dz \right] \tag{66}$$

This correctly yields $\tau_a = 0$ at $z = z_o$ when $Q(z_a)$ is determined. In the vicinity of $z_o$, let $A = A_1(z_o - z)$.[2] Then from equation (26) and Table 1,

$$A_1(z_o) = a_o G_1 G_3 G_8 G_{12} V + G_0 G_2 G_6 G_7 \tag{67}$$

Then $Q(z_o)$ can be evaluated as

$$Q(z_o) = \frac{1}{\sqrt{G_5 G_4^3 \sqrt{A_1}}} \tag{68}$$

[2] The area $A$ is linear in $z$ for a uniform atmosphere.

In the digital program, the age is calculated in subroutine OUTPU from equation (66) using equations (65), (67), and (68) and a Simpson integration procedure for the integral. It is listed with the ray-tube area and ray coordinates as a function of altitude z .

The phase variable $\xi_1$ is calculated at the ground $(z = 0)$ in subroutine OUTPR and is listed in the output table.

Shock Location.- The integral $S_o(\xi)$ (eq.(54)) and the function $\mathcal{S}(\xi_1)$ (eq. (55)) are obtained in subroutine OUTPR. Values of these variables are listed in the output table at corresponding values of $\xi$ and $\xi_1$ , where SINT is $S_o(\xi)$ and S is $\mathcal{S}(\xi_1)$ . The reflection factor $K_R$ , input on Card 10, is used to multiply the pressure, also in subroutine OUTPR. The resulting pressure increment $\Delta p$ (eq. (57)) and pressure ratio $p_1 = \Delta p/p$ (where p is ambient pressure) are listed as DELP and P1, respectively, in this output table.

As described in THEORETICAL ANALYSIS, Shock Location, the user must locate the shocks and the shock jumps by plotting $\mathcal{S}(\xi_1)$ and $\Delta p$ versus $\xi_1$ given in this output table.

## Conversion to Other Computers

The program listing of Appendix B is the program used for computations on the CDC-6600. The following remarks are pertinent to conversion to other computers:

1. Input and output units are standard FORTRAN units 5 and 6. For other units, change KUNIT = 6 and LUNIT = 6 in subroutine INPU. If a different working tape than tape unit 8 is used (saves ground intersection data), change MUNIT = 8 in INPU.

2. Some computers use FORMAT statements requiring the hollirith notation ' , whereas the symbol * is used here for the CDC-6600.

3. To fit within the core storage limitations of the IBM-1130, links, locals and socals have been required.

4. Monitor control cards must be added to be compatible with the compiler.

5. The sense switch subroutine SSWTCH should be checked for acceptability.

6. The dimension statements pertaining to the tabular input data may need to be changed to accommodate larger tables.

## COMPUTATION RESULTS

### Sample Printout

Appendix B presents a reproduction of a complete printout for a typical solution. Various sections of the printout are delineated on the right-hand side by letters A through E.

Section A is the input data which is listed for identification and verification. The number of significant figures in the listing might be less than those on the input data cards and stored within the computer. These inputs represent an airplane flying in an easterly direction ($\psi = 90^{o}$) at 30 000 ft (9.1 km) in a climbing turn near Mach 1.5. The ground altitude is 1500 ft (457 m). An F-function is tabulated for two ray azimuth angles, $\phi = 0$ and $45^{o}$, and for 19 stations along the length of the aircraft. The F-function table differs for the two $\phi$'s only at station L/LA = 0.60. The wind direction is from the west ($\eta = 270^{o}$, a tail wind), but with zero speed. The inputs are in Special units (IUNIT = 1), and the atmosphere is the 1962 U.S. Standard (ISTAN = 1). Rays are to be initiated at the aircraft at one-second intervals of flight time (DELTA T PRINT). Data are to be computed for rays initiated at azimuth angle intervals of $20^{o}$ (DPHI) to a maximum value of $\pm 60^{o}$ (MAXIMUM PHI) using an initial integration interval of approximately 300 ft (91 m) (DELTA Z). Ray-tube information is requested at 2000 ft (610 m) intervals of altitude (PRINT INTERVAL) and at the next integration interval below 3500 ft (1.1 km) (PRINT OUT POINT). The ground reflection factor $K_R$ which multiplies the pressure increment is 1.8 whereas the F-function multiplier $F_f$ is 3.784 (eq. (34)).

Section B illustrates Maneuver Data output. These include the state variables of the aircraft at the time $t_a$ when the ray tracing is started. Force coefficients, wind components, and dynamic pressure are also listed. Maneuver Option 1 was selected (IM = 1), so the derivatives $\dot{V}$ , $\dot{\gamma}$ , and $\dot{\psi}$ were calculated making use of the load factor data in the input section ($n_T$, $n_L$).

Section C represents the data listed during propagation of the signal from the aircraft to the ground. The first line identifies the particular azimuth angle $\phi$ and the values of $\nu$, $c_{oo}$ and $c_o$ at the aircraft altitude. Below this is a table of ray location, time, ray-tube area, age, and $\cos\theta$ as functions of altitude z.

Section D is the pressure signal data at the ground. Data are given for each integration interval of the $\mathcal{S}$-curve. It is important to have a large set of points so that the pressure and $\mathcal{S}$-curves can be plotted accurately. In this table, F is the

input value $F_i$ multiplied by $F_f$, $V_E(L)$ is the signal invariant (eq. (45)), XI 0 is the phase $\xi$, XI 1 is the phase $\xi_1$, S INT is the integral of $V_E(\xi)$, and S is the value of the function $\mathcal{S}$ from which shock locations are determined. The pressure parameters $p_1 = \Delta p/p$ and $\Delta p$ are listed in the last two columns.

Both $\Delta p$ and $\mathcal{S}$ are to be plotted against the independent variable $\xi_1$ as shown in figure 15 for a $M = 1.2$ solution.

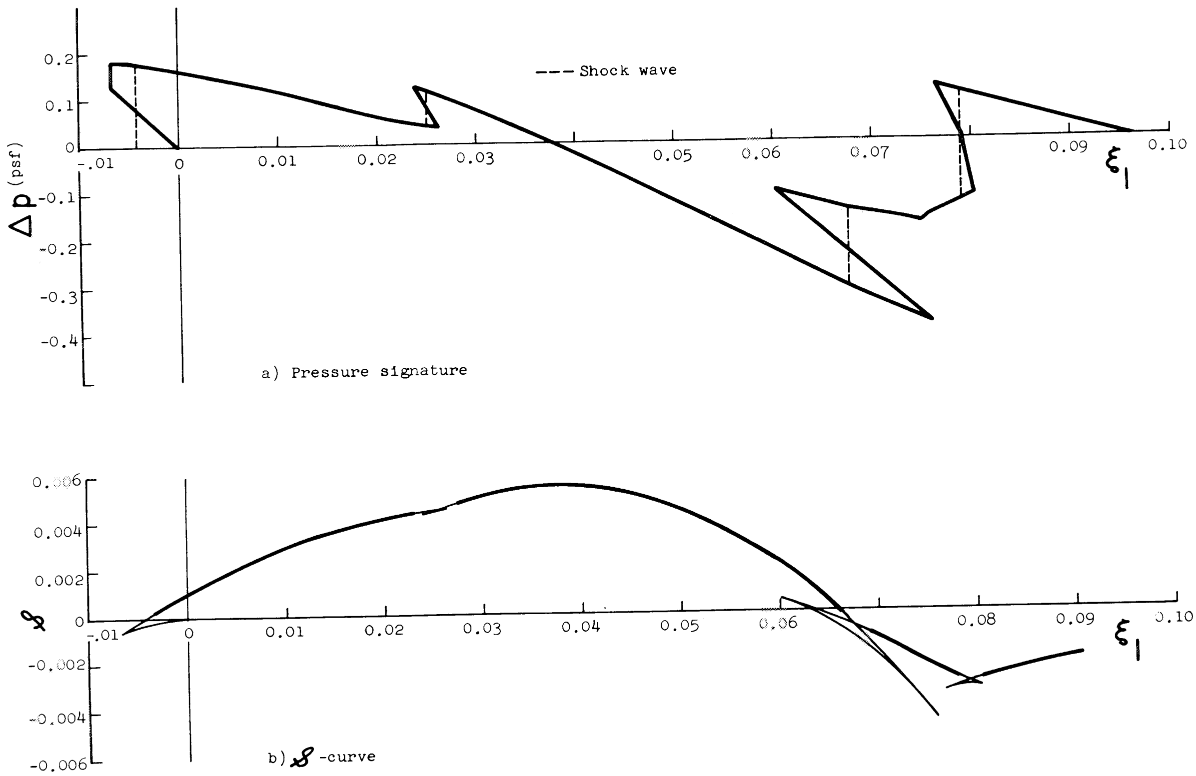



Figure 15. Distorted pressure signature and $\mathcal{S}$-curve at ground; standard atmosphere, $M = 1.2$

The shock locations are defined wherever $\mathcal{S}$ crosses itself when proceeding along the curve with increasing $\xi_1$ on the uppermost branches or boundary (sup $\mathcal{S}$). The first crossing is on the abscissa ($\xi = -0.004$ in the example of figure 15) because $\mathcal{S}$ originates at $-\infty$ with value zero until $\xi_1 = 0$. The pressure jumps at the shock waves are shown by dashed lines. These represent the correct profile of the pressure where the solution would otherwise appear to be multivalued.

Section E presents the listing for the ground intersection curves. These are the interpolated values of ray location and peak pressure for constant arrival time at the ground.

## Typical Results

Various solutions have been calculated to check the equations which were programmed and to compare a few sample results with previous analyses. Two characteristics which can be evaluated analytically to provide checks on many factors and terms in the equations, but excluding effects of winds, are

a) Rays travel in a circular path (ref. 38) in an atmosphere which has a speed of sound varying linearly in $z$.

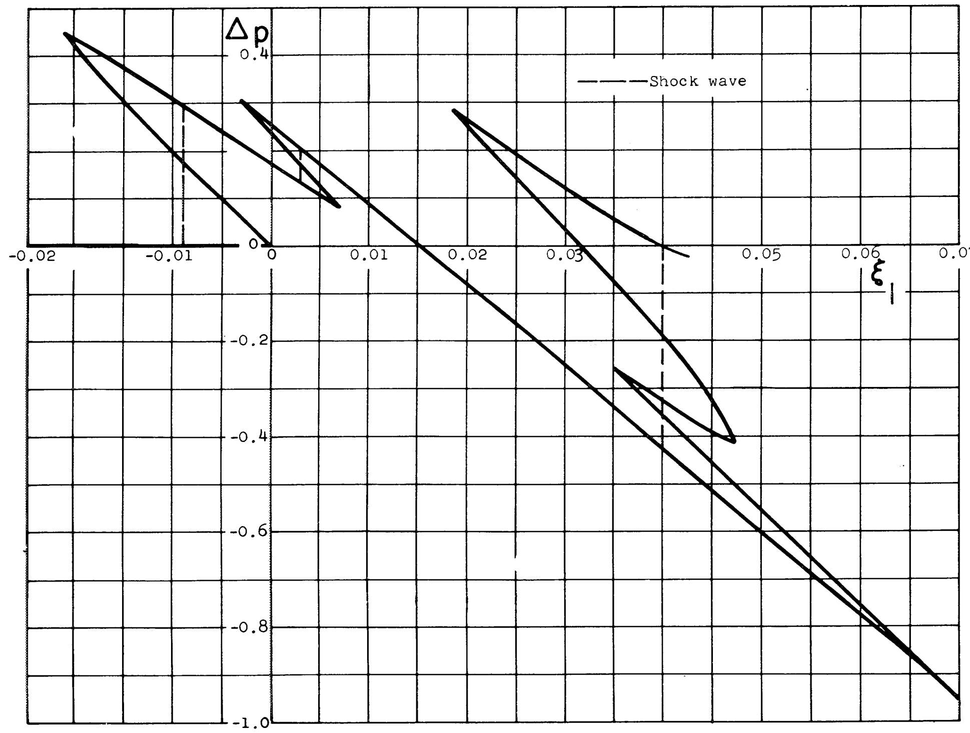
b) Ray-tube area which is computed for an atmosphere with a constant temperature will increase quadratically in the lateral distance parameter ($ax^2 + bx + c$, if the aircraft is traveling south). Moreover, if the aircraft is in steady flight (time derivatives all zero), the area will increase proportionately to the decrease in $z$.

The present program accurately gave these analytic results for representative examples.

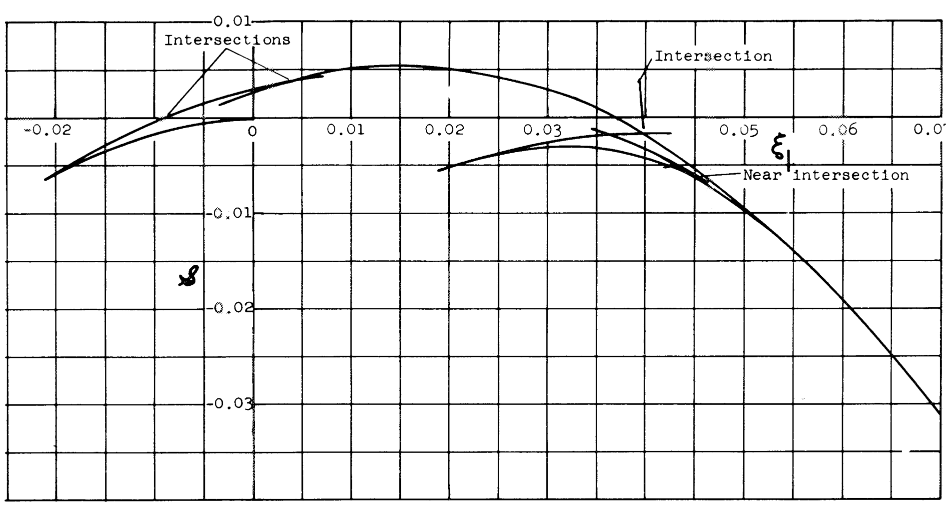
In addition, solutions for typical practical situations have been calculated to demonstrate the program using the F-function of Appendix B, Section A. Two examples are shown in figures 15 and 16 for steady flight at $M = 1.2$ and $M = 3$, respectively, at 50 000 ft (15.2 km) altitude and for the ground intersection directly below the aircraft ($\phi = 0$). In comparison with the data for $M = 1.2$, both the $\mathscr{J}$-curve and the pressure curve reflect the greater age of the $M = 3$ signature, as the pressure profile has developed nearly to an N-wave solution. Indeed, the two shocks on the right for $M = 1.2$ have merged into only one at $M = 3$.

Results using an atmosphere which has a larger lapse rate than standard, with a temperature of 90°F at the ground and -75°F at 30 000 ft (9.1 km), are shown in figure 17 for several Mach numbers. This atmosphere is designated ATM A-3 in reference 48. The pressure ratio here is the maximum pressure listed for the nonstandard atmosphere divided by the maximum pressure in a standard atmosphere.

Tailwind effects on signal strength are summarized in figure 18 for some initial calculations. The wind is the mean zonal of figure 19 (see ref. 48). The pressure ratio here is the maximum pressure listed for the tailwind solution divided by the corresponding value in a standard no-wind atmosphere. A headwind would give pressure ratios larger than 1.0 near Mach 1.2 and represents a more serious sonic boom environment.




a) Pressure signature




b) $\mathscr{S}$-curve

Figure 16. Distorted pressure signature and $\mathscr{S}$-curve at ground; standard atmosphere, M = 3

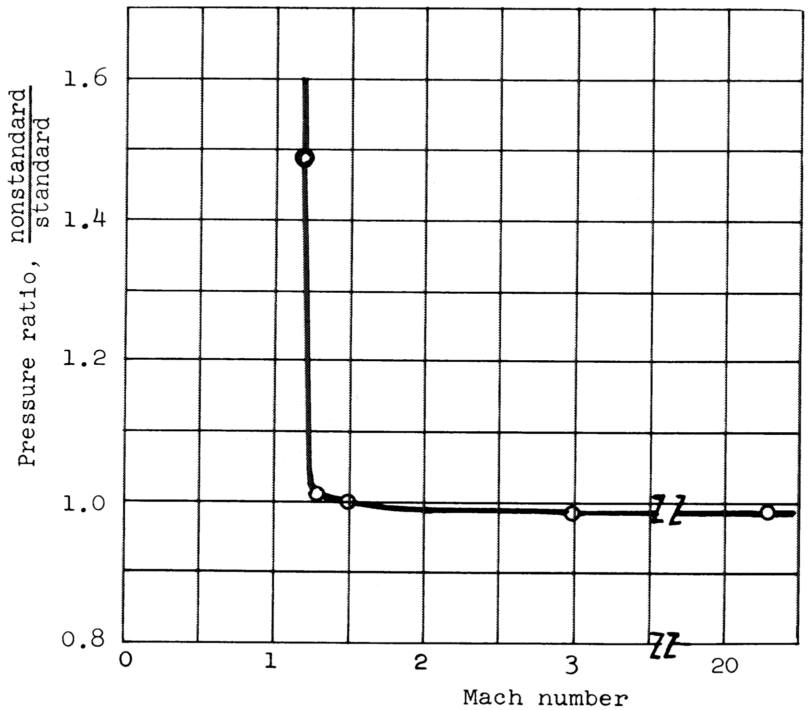



Figure 17. Temperature effects on sonic boom

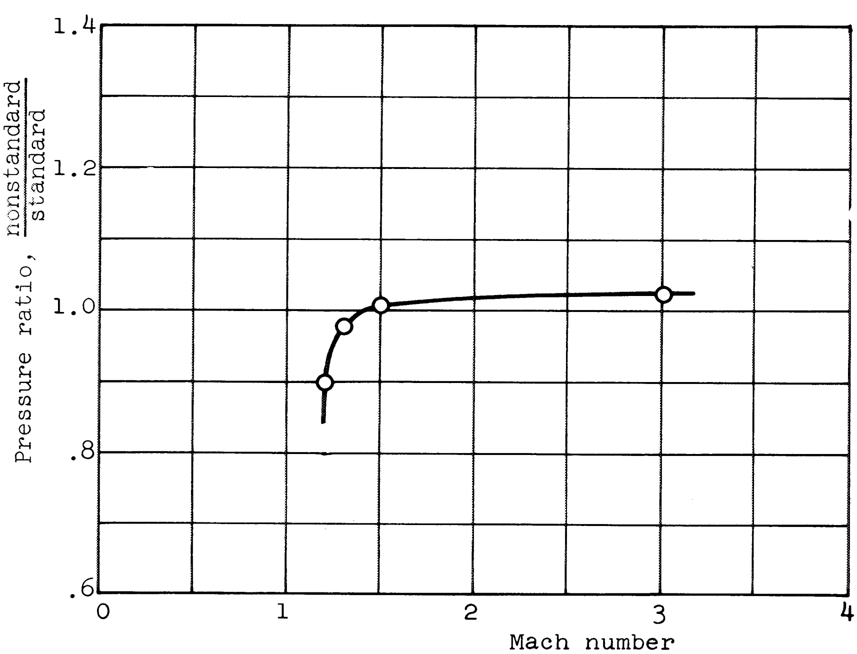



Figure 18. Tailwind effects on sonic boom

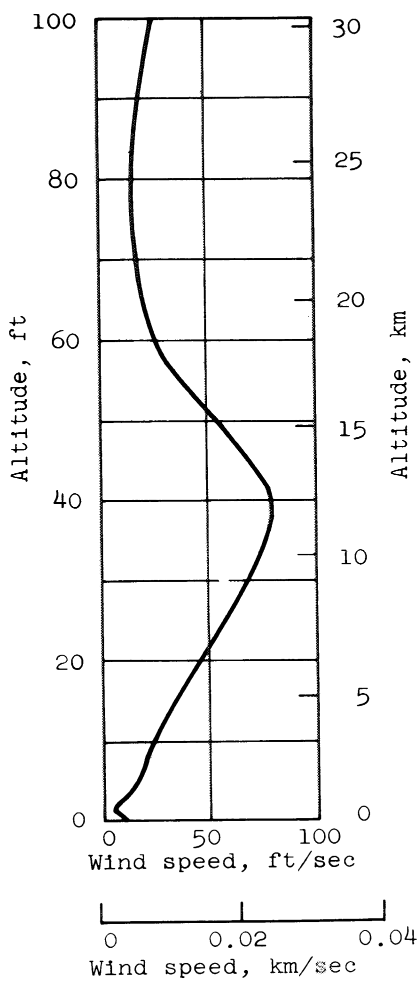



Figure 19.
Mean zonal wind

The preceding results were for rays starting directly beneath the aircraft, $\phi = 0$ . For increasing $\phi$ , the maximum lateral extent of ray intersections with the ground is shown in figure 20 for a standard atmosphere. The effect of wind on lateral range is shown in figure 21. It is seen that lateral range is considerably reduced by a headwind. Results with a nonstandard atmosphere are shown in figure 22 for atmosphere B-5 of reference 48 (multiple temperature inversions in troposphere).

The wind and temperature effects brought out by these examples on sonic boom intensity and maximum lateral range are similar to those presented in more detail in reference 48. Specifically, only small differences from standard atmosphere, no wind solutions are indicated at aircraft speeds larger than $M = 1.3$ .

The results of the sample problem listed in Appendix B provide an example of a solution with an airplane maneuver. For an axial acceleration of 0.2 g's and a lift acceleration of 0.8 g's in a turn, the maximum pressure is 61% greater than for nonaccelerating straight flight at Mach 1.5. Thus, while atmospheric variations may be expected to be essentially unimportant to sonic boom intensity at speeds above $M = 1.3$ , maneuver effects can continue to have a strong effect.

## CONCLUDING REMARKS

This report has presented an analysis of sonic boom propagations in a stratified atmosphere with winds, and a computer program based on the analysis. The analysis and the program take into account maneuvers of the aircraft and yield actual pressure signatures without the common N-wave approximations. Some additional theoretical discussion and historical notes have been included to make the report more useful as an exposition of the fundamental theory. Sample computer input and output data are presented with results of some preliminary calculations made using the program.

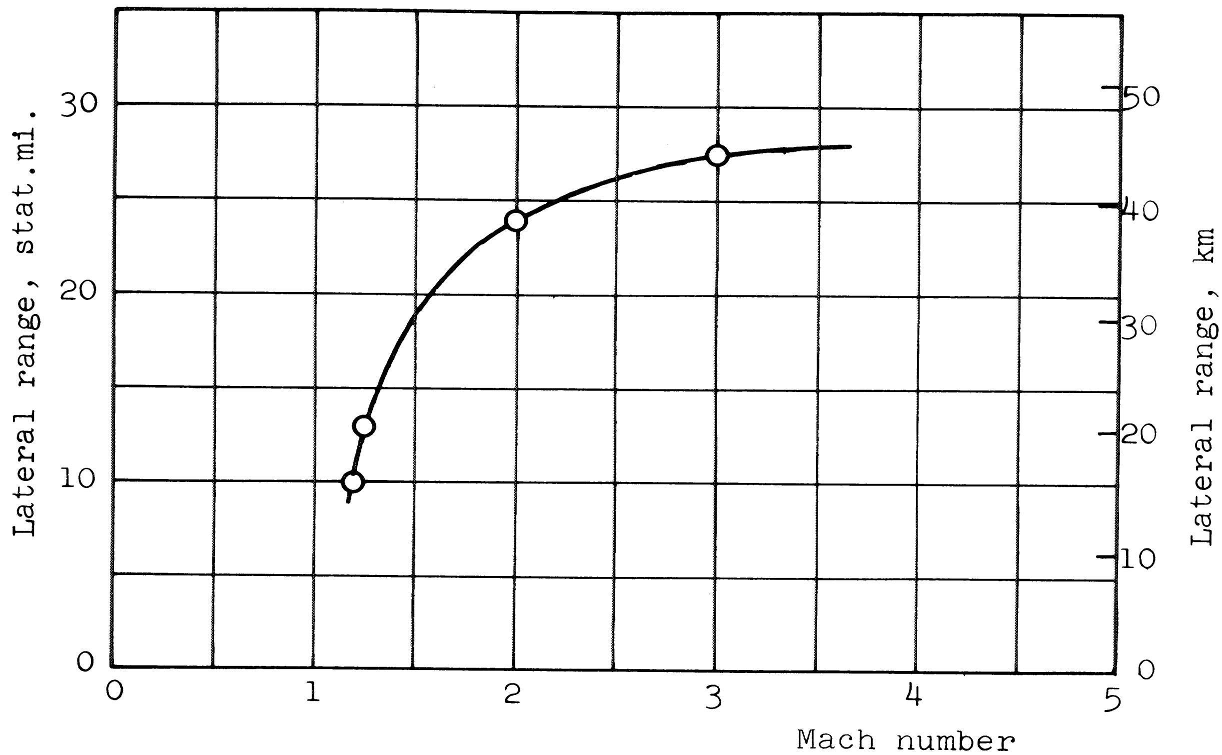



Figure 20. Lateral range of sonic boom cutoff, standard atmosphere, h = 50 000 ft (15.24 km)

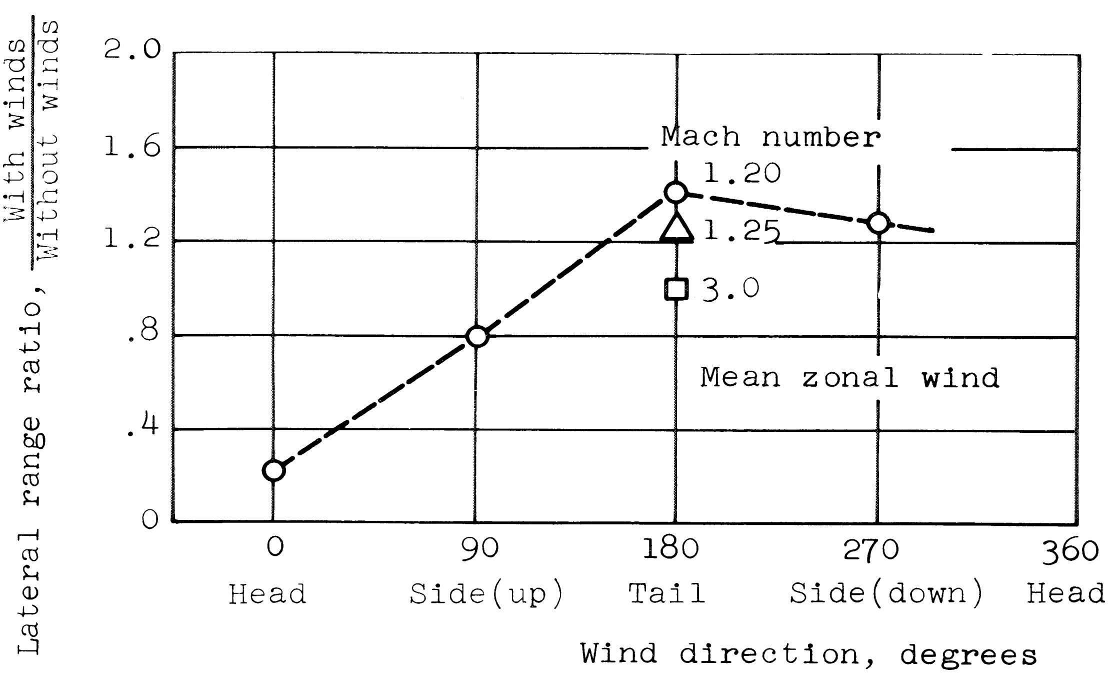



Figure 21. Wind effects on maximum lateral range

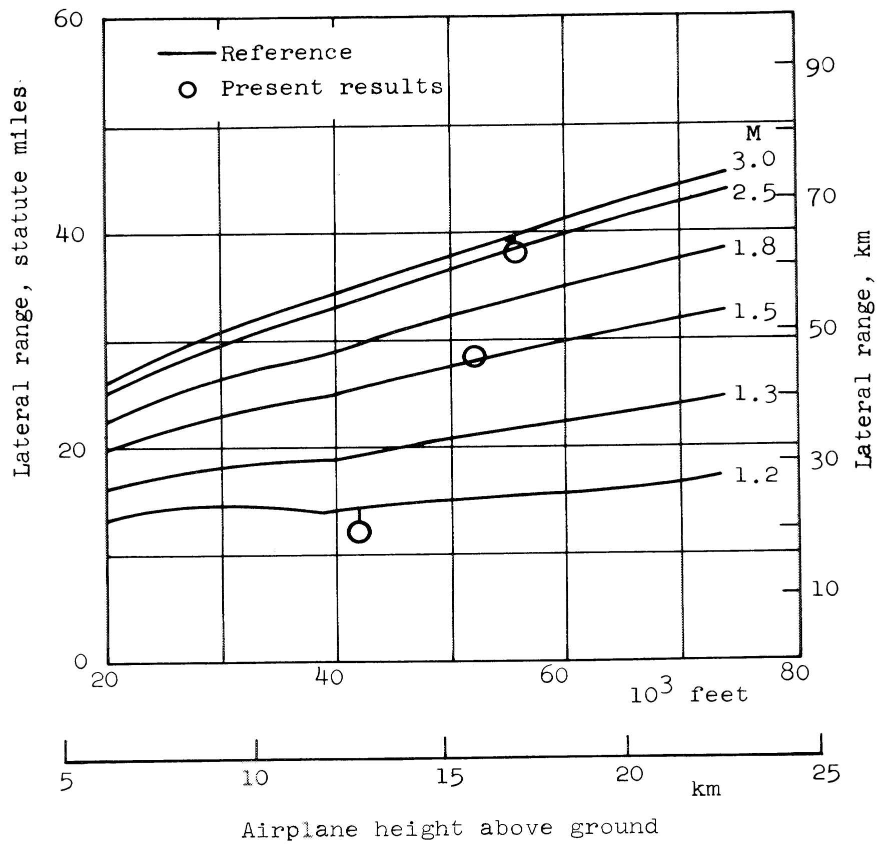



Figure 22. Lateral range with atmosphere B-5

The digital program was written to allow a number of input options and to be readily adaptable to various computers. Complete definitions and instructions for its use are presented.

Preliminary results have been shown which indicate the capabilities of this new program. It can be applied to analyzing the effects on sonic boom strength of changes in atmospheric conditions, wind profiles, and aircraft maneuvers. It is, therefore, useful to the aircraft designer to show effects of aircraft geometry and initial signature, to the environmental science services (meteorologists) to show significance to atmospheric variations for specific aircraft, and to the governmental aviation authorities and the airlines operations analysts to provide a basis for specifying permissible flight maneuvers and flight profiles within sonic boom overpressure constraints.

It is possible and desirable, of course, to make certain additions and alterations in the future to the program. One of

these may be to provide equations at the beginning of the program for directly calculating the aircraft F-function, now required as a known input. Another is to extend the automatic calculations to provide directly the $\mathcal{S}$-curve intersections (shock locations) and shock strengths, and to provide automatic plotting of the final pressure signatures. These are now manual operations. The present program, in common with other known techniques, cannot provide pressure information when a caustic develops, nor can it account for atmospheric turbulence. These are two important areas requiring further research.

## REFERENCES


1. Hayes, W.D.: Review of Sonic Boom Theory. Proc. AFOSR-UTIAS Symposium on Aerodynamic Noise. Univ. of Toronto Press, 1968.

2. U.S. Standard Atmosphere, 1962. U.S. Government Printing Office, Washington, D.C. 1962.

3. Friedman, M.P.: A Description of a Computer Program for the Study of Atmospheric Effects on Sonic Boom. NASA CR-157, 1965.

4. Hayes, W.D.: Geometric Acoustics and Wave Theory. Second Conference on Sonic Boom Research, NASA SP-180, 1968.

5. Groves, G.V.: Geometric Theory of Sound Propagation in the Atmosphere. J. Atmos. Terr. Phys., vol. 7, 1955, pp. 113-127.

6. Rayleigh, Lord (J.W. Strutt): The Theory of Sound, Vol. II. Macmillan Co., 1878. Reprint of 2nd ed., Dover Publications, New York, 1945.

7. Barton, E.H.: On the Refraction of Sound by Wind. Phil. Mag. (6th Series), vol. 1, 1901, pp. 159-165.

8. Fujiwhara, S.: On the Abnormal Propagation of Sound Waves in the Atmosphere. Bull. Central Meteo. Observ. of Japan, vol. 2, no. 1, 1912.

9. Emden, R.: Beiträge zur Thermodynamik der Atmosphäre, Vol. II. Über die Ausbreitung des Schalles in einer Windbewegten Polytropen Atmosphäre. Meteorol. Zeit., vol. 35, 1918, pp. 13-29.

10. Lighthill, M.J. Group Velocity. J. Inst. Math. Applic., vol. 1, 1965, pp. 1-28.

11. Chester, W.: Supersonic Flow past Wing-body Combinations. Aeron. Quart., vol. 4, 1953, pp. 287-314.

12. Hayes, W.D.: Linearized Supersonic Flow. Thesis, Calif. Inst. of Tech., 1947. Also North Amer. Aviation, Inc., Report No. AL-222, 1947.

13. Lomax, H.: The Wave Drag of Arbitrary Configurations in Linearized Flow as Determined by Areas and Forces in Oblique Planes. NACA RM A55A18, 1955.

14. Lomax, H.; and Heaslet, M.A.: Recent Developments in the Theory of Wing-Body Wave Drag. J. Aero. Sci., vol. 23, 1956, pp. 1061-1074.

15. Walkden, F.: The Shock Pattern of a Wing-Body Combination Far from the Flight Path. Aeron. Quart., vol. 9, 1958, pp. 164-194.

16. Whitham, G.B.: On the Propagation of Weak Shock Waves. J. Fluid Mech., vol. 1, 1956, pp. 290-318.

17. Whitham, G.B.: The Flow Pattern of a Supersonic Projectile. Comm. Pure. Appl. Math., vol. 5, 1952, pp. 301-348.

18. Keller, J.B.: Geometric Acoustics. I. The Theory of Weak Shock Waves. J. Appl. Phys., vol. 25, 1954, pp. 938-947.

19. Guiraud, J.-P.: Acoustique géométrique, bruit ballistique des avions supersoniques et focalisation. J. de Mecanique, vol. 4, 1965, pp. 215-267.

20. Blokhintsev, D.I.: Acoustics of a Nonhomogeneous Moving Medium. Gostekizdat, 1946. NACA TM 1399, 1956.

21. Blokhintsev, D.I.: The Propagation of Sound in an Inhomogeneous and Moving Medium, Part I. J. Acoust. Soc. Am., vol. 18, 1946, pp. 322-334.

22. Ryzhov, O.S.; and Shefter, G.M.: On the Energy of Acoustic Waves Propagating in Moving Media. Appl. Math. Mech. (PMM), vol. 26, 1962, pp. 1293-1309.

23. Chernov, L.A.: Flow and Density of Acoustic Energy in a Moving Medium. Zh. Tekh. Fiz., vol. 16, 1946, pp. 733-736. Appl. Phys. Lab., Library Bull. TG-230-T291, 1962.

24. Garrett, C.J.R.: Discussion: The Adiabatic Invariant for Wave Propagation in a Nonuniform Moving Medium. Proc. Roy. Soc. (London), Ser. A, vol. 299, 1967, pp. 26-27.

25. Hayes, W.D.: Energy Invariant for Geometric Acoustics in a Moving Medium. Phys. Fluids, vol. 11, 1968, pp. 1654-1656.

26. Bretherton, F.P.; and Garrett, C.J.R.: Wavetrains in Inhomogeneous Moving Media. Proc. Roy. Soc. (London), Ser. A, vol. 302, pp. 529-554.

27. Hayes, W.D.: Pseudotransonic Similitude and First-Order Wave Structure. J. Aeron. Sci., vol. 21, 1954, pp. 721-730.

28. Whitham, G.B.: A New Approach to Problems of Shock Dynamics. Part I. Two-dimensional Problems. J. Fluid Mech., vol. 2, 1957, pp. 145-171.

29. Whitham, G.B.: A New Approach to Problems of Shock Dynamics. Part II. Three-dimensional Problems. J. Fluid Mech., vol. 5, 1959, pp. 369-386.

30. Poisson, S.D.: Mémoire sur la théorie du son. J. de l'École Polytecnique, vol. 7 (cahier 14), 1808, pp. 319-392.

31. Earnshaw, S.: On the Mathematical Theory of Sound. Phil. Trans. Roy. Soc. London, vol. 150, pp. 133-148.

32. Chandrasekhar, S.: On the Decay of Plane Shock Waves. Ballistic Res. Labs., Aberdeen, Md., BRL Rept. No. 423, 1943.

33. Landau, L.D.: On Shock Waves. J. Phys., USSR, vol. 6, 1942, pp. 229-230.

34. Landau, L.D.; and Lifshitz, E.M.: Sound Waves in Second Approximation. Sect. 78 of: Mechanics of Continuous Media. Gostekhizdat, 1944. Essentially the same as: Formation of Discontinuities in a Sound Wave. Sect. 95 of: Fluid Mechanics. Pergamon Press, 1959.

35. Rao, P.S.: Supersonic Bangs. Aeron. Quart., vol. 7, 1956, pp. 21-44, 135-155.

36. Whitham, G.B.: The Propagation of Weak Spherical Shocks in Stars. Comm. Pure Appl. Math., vol. 6, 1953, pp. 397-414.

37. Hayes, W.D.: Viscous First-order Wave Structure (abstract). Actes, IXe Congrès Inter. de Méc. Appl., Univ. de Bruxelles, Vol. III, 1957, p. 346.

38. Hayes, W.D.: Long-range Acoustic Propagation in the Atmosphere. Inst. for Defense Analyses, Jason Div., Res. Paper P-50, 1963.

39. Busemann, A.: The Relation between Minimizing Drag and Noise at Supersonic Speeds. Conf. on High-Speed Aeronautics. Polyt. Inst. of Brooklyn, 1955, pp. 133-144.

40. Burgers, J.M.: Further Statistical Problems Connected with the Solution of a Simple Nonlinear Partial Differential Equation. Proc. Kon. Nederlandse Akad. van Wet., Series b, Vol. 57, 1954, pp. 159-169.

41. Crussard, L.: Sur la propagation et l'altération des ondes de choc. C. R. Acad. Sci. Paris, vol. 156, 1913, pp. 611-613.

42. Hopf, E.: The Partial Differential Equation $u_t + uu_x = \mu u_{xx}$. Comm. Pure Appl. Math., vol. 3, 1950, pp. 201-230.

43. Landau, L. D.: On Shock Waves Far from their Source. Prikl. Mat. Mekh., vol. 9, 1945, pp. 286-292. Collected Papers of L. D. Landau, Gordon and Breach, 1965, pp. 437-444.

44. Lighthill, M. J.: Viscosity Effects in Sound Waves of Finite Amplitude. Surveys in Mechanics (70th anniv. of G. I. Taylor; eds. G. K. Batchelor and R. M. Davies), Cambridge, 1956, pp. 250-351.

45. Middleton, W. D.; and Carlson, H. W.: A Numerical Method for Calculating Near-Field Sonic Boom Pressure Signatures. NASA TN D-3082, 1965.

46. Taylor, G. I.: The Conditions Necessary for Discontinuous Motion in Gases. Proc. Roy. Soc.(London), Ser. A, Vol. 84, (911), pp. 371-377.

47. Hayes, W. D.: Gasdynamic Discontinuities. Part D of: Fundamentals of Gas Dynamics (H. Emmons, ed.). Princeton U. Press, 1958, pp. 416-481. Princeton Aeronautical Paperback no. 3, Princeton U. Press, 1960.

48. Kane, Edward J.; and Palmer, T.Y.: Meteorological Aspects of the Sonic Boom. FAA SRDS Report No. RD-64-160, September 1964 (Available from DDC as AD-610463).

TABLE 1.

NAMES OF VARIABLES IN PROGRAM

COMMON

| | |
|---|---|
| A | Speed of sound |
| AO | A at the initial point of ray |
| ACC<br>ACCUR<br>PHSIG | Accuracy criteria for integrals evaluated using NODE |
| AGE | Age variable $\tau$ |
| ALT | Altitudes in altitude/temperature input table |
| ARA | Area of ray tube |
| BETA | $\sqrt{M^2 - 1}$ |
| BYHAL<br>REDUC | Halving suppression switch for integration |
| C | Coefficients for Simpson's evaluation of AGE |
| CN | $\cos \eta$ |
| CON1 | $\sqrt{G50 \cdot \cos^2\theta_o / \tan \mu}$ } also used as coefficients in AGE calculation |
| CON2 | $\sin \mu / c_o \cdot \cos \theta$ } also used as coefficients in AGE calculation |
| CON3 | AGE/2 |
| CON4 | $(G50/G4 \cdot ARA)^{1/2}$ |
| COO | $a_o / \cos \theta_o$ |
| CO | $c_{oo} - u_{no}$ |
| CT | $\cos \theta$ |
| CTH | $\cos \theta_o$ |
| CU | $\cos \mu$ |
| D | Atmospheric density for a given altitude z |

| | |
|---|---|
| DADZ | da/dz |
| DEL | Current step in $\phi$ |
| DELFI | Angle increment for $\phi$ |
| DELP | Maximum and initial step size for ray-area calculation |
| DELPT | Print interval for ray-area calculation |
| DELZ1 | Maximum initial step size for ray-area calculation |
| DPMAX | Maximum value of pressure for all F's at a specified $\phi$ |
| DSIM | Step for Simpson's rule integration for AGE |
| DTPRN | Print interval for maneuver calculation |
| EETA | Wind direction for a given z |
| ENDNO | Number of points in integration range |
| ENDVA<br>ENDVL | Value of independent variable at end of integral |
| ETA | Wind direction in input table |
| ETADZ | $d\eta/dz$ |
| F | Two-dimensional array of F-function input |
| FA | Bank angle at initial point of ray |
| FIA | Bank angle in input table |
| FIMAX | Maximum value of $\phi$ |
| FI | Ray azimuth angle $\phi$ measured from vertical plane |
| FLAG | Set to -1 to terminate integration |
| GAMR | Flight path angle $\gamma$ in input table |
| GDOT | $d\gamma/dt_a$ |
| GR | Value of $\gamma$ for a given time |
| GO | $\cos\theta/\sin\theta$ |

| | | |
|---|---|---|
| G1 | $GO^3/a^2$ | |
| G2 | $1 + (\sin\gamma/\sin\mu \sin\theta_o)$ | also used as temporaries |
| G3 | $\cos\gamma \cos\mu \sin\phi/\cos\theta_o$ | also used as temporaries |
| G4 | $\sin\theta \cos\theta$ | |
| G5 | $\rho a^2$ | |
| G50 | $\rho_o a_o^2$ | |
| G6 | $\cos\mu/\cos^2\theta_o$ | |
| G7 | $\cos\mu \sin\gamma + \sin\mu \cos\gamma \cos\phi$ | |
| G8 | $\sin\theta_o/\cos^3\theta$ | |
| G9 | $\cos\mu \sin\theta_o \sin\phi/\cos^2\theta_o$ | |
| G10 | $\sin\theta_o \sin\phi$ | |
| G11 | $\cos\gamma \sin\phi/\cos^2\theta_o$ | |
| G12 | $\cos\mu \cos\gamma \sin\phi$ | |
| G13 | $\sin\mu \cos\gamma + \cos\mu \sin\gamma \cos\phi$ | |
| H<br>HNODE | Step size used for integration | |
| HG | Height of ground above sea level | |
| HW | Height for input wind table | |
| HO | Initial value of aircraft height above sea level | |

H1, H2, H3, H4, H5, H7: Variables representing terms and factors in equation for ray-tube area:

H1
$$= u_{to}\frac{d\psi}{dt_a} - \left(a_o G_8 G_7 - u_{to} G_{11}\right)\frac{d\mu}{dt_a}$$
$$- \left(a_o G_8 G_{13} - u_{to} G_9\right)\frac{d\gamma}{dt_a}$$
$$- \left(a_o \sin\gamma_r/\sin\mu\right)\left(u'_{no} - a'_o/\cos\theta_o\right)$$

| | | |
|---|---|---|
| H2 | | $= \frac{d\psi}{dt_a} + G_{11} \frac{d\mu}{dt_a} + G_6 G_{10} \frac{d\gamma}{dt_a}$ |
| H3 | | $= - u_{to} G_6 G_7 - a_o G_8 G_{12}$ |
| H4 | | $= - G_6 G_7$ |
| H5 | | $= G_3 V + u_{to} G_2$ |
| H7 | | (H1)(H4) = (H2)(H3) |
| I | | Counter for Simpson's rule and various DO loops |
| IALT | | Altitude geopotential/geometric switch |
| IM | | Maneuver option 1 or 2 |
| IOUT | | Switch for first z printout |
| IPRIN | | Switch for printing at a selected z |
| IRH | | Switch to indicate if ray is horizontal or area is zero |
| IRHO | | Altitude, temperature, pressure/density switch |
| ISTAN | | Standard/tabular atmosphere switch |
| ITEMP | | Type of temperature units input |
| IUNIT | | English/metric/special units switch |
| KTIME | | Number of ground intersections stored |
| L | | DO instruction parameter for steps in S-integral |
| LAMDO | | Value of $\psi$ for a given value of time |
| LAMR | | Input table of heading angles ($\psi$) |
| LA | | Length of airplane |

| | |
|---|---|
| LUNIT | Output unit number |
| M | Mach number input table |
| ML | Position in PHI table for F-function |
| MM | Mach number for a given time |
| MTIME | Switch to indicate if first maneuver step |
| MUDOT | $d\mu/dt_a$ |
| MUNIT | I/O unit on which ground data are saved |
| MU | Mach angle $\mu$ |
| N<br>NNODE | Number of integrals to be evaluated simultaneously |
| NALTS | Number of entries in the altitude/temp/density table |
| NEW | Heading angle $\nu$ of wave propagation vector |
| NIM | Number of entries in maneuver table |
| NL | Load factor input table, lift direction |
| NODUB | Switch to suppress doubling step during integration |
| NODUM | Switch to suppress printing of bad points during integration |
| NPHI | Number of $\phi_r$'s for the F-function table |
| NSAVE | Number of auxiliary variables calculated during integration |
| NT | Load factor input table, axial direction |
| NTAU | Number of F-functions for each $\phi$ |
| NWIND | Number of entries in wind table |
| P | Pressure for a given $z$ |
| PHI | Input table of $\phi_r$'s for which F-values are tabulated |
| PRESS | Pressure input table |

| | |
|---|---|
| PRIN | Selected value of $z$ at which additional printing is desired |
| PSIDT | $d\psi/dt_a$ |
| RE | Radius of the earth |
| RF | Reflection factor |
| RHO | Atmospheric density input table |
| RHOO | Density at the initial point of ray |
| SN | sin $\nu$ |
| ST | sin $\theta$ |
| STH | sin $\theta_o$ |
| SU | sin $\mu$ |
| SUM | Used in collection of terms for AGE calculation |
| T, TNODE | Used in integration step size determination for NODE |
| TAU | Length ratio L/LA in input table |
| TEMP | Temperature input table |
| TIME | Time at initial point of ray |
| TIND | Time at current integration point in maneuver |
| TINIT | Time at which maneuver is to be started |
| TLAS | Last value of time at which interpolation was made |
| TPRIN | Next value of time for a maneuver printout |
| TP | Temporary location |
| TT | Input table of times for maneuver table |
| UNO | Component of wind speed in direction of wave propagation |
| UTO | Component of wind speed transverse to wave propagation |

| | |
|---|---|
| V | Speed of the airplane |
| VAR | Two-dimensional array used to save back points in integration |
| VVW | Input table of wind speeds |
| VWX | X-component of wind at maneuver point |
| VWXX | X-component of wind |
| VWY | Y-component of wind at maneuver point |
| VWYY | Y-component of wind |
| VW | Value of wind speed at a given z |
| WS | Wing loading |
| XM | x at initial point of ray (a maneuver point) |
| YM | y at initial point of ray |
| Z | z at current point of ray |
| ZINIT | z at initial point of ray |
| ZPRIN | Next value of z to be printed |
| ZSIMP | Value of z for next Simpson's step |

<u>INPU</u>

| | |
|---|---|
| FF | Factor to rescale F table |
| I | DO instruction parameter |
| J | DO instruction parameter |
| KUNIT | I/O unit for input |

<u>ATMO</u>

| | |
|---|---|
| D | Density at height z |

| | |
|---|---|
| H | $z$ plus height of ground |
| HGLTB | Array for base heights |
| HGP | H in table value units (geopotential meters) |
| I | DO instruction parameter |
| J | Layer number being used |
| K1 | Array of $K_1$ coefficients |
| K23 | Array of $K_{23}$ coefficients |
| P | Pressure at height $z$ |
| PB | Array of $P_B$ coefficients |
| RHOB | Array of $\rho_B$ coefficients |
| TB | Array of $t_B$ coefficients |
| TEM1 | Difference between HGP and table value |
| TEM2 | Quantity used in calculation of D |
| TH | Temperature at height $z$ |
| THDZ | dTH/dz |
| VS | Speed of sound at height $z$ |
| Z | Current height above ground |

<u>ANGLE</u>

| | |
|---|---|
| ARRAY | Array to be checked |
| I | DO instruction parameter |
| MAX | Number of entries in ARRAY table |
| MAXS | MAX - 1 |

DERM2

| | |
|---|---|
| BLANK | Dummy variable for LAGRA call |
| COSGA | cos $\gamma$ |
| COSLA | cos $\psi$ |
| J | Sense switch value |
| SINLA | sin $\psi$ |
| TH | Temperature from standard atmosphere program |
| TM | Temperature at a given $z$ |

OUTM2

| | |
|---|---|
| BLANK | Dummy variable for LAGRA call |
| CL | Lift coefficient |
| COSDI | cos $(\psi - \eta)$ |
| COSGA | cos $\gamma$ |
| COSMU | cos $\mu$ |
| C1 | Axial force coefficient |
| NLZ | Value of NL at point $z$ |
| NTZ | Value of NT at point $z$ |
| Q | Dynamic pressure |
| SINDI | sin $(\psi - \eta)$ |
| SINGA | sin $\gamma$ |
| TERM1 | Subterm in derivative calculations |
| TH | Temperature at $z$ from standard atmosphere program |
| THDZ | dTH/dz by quadratic interpolation |
| TM | Temperature for a given $z$ value |

| | |
|---|---|
| VDOT | $dV/dt_a$ |
| VG | Velocity of aircraft relative to ground |

<u>INPOL</u>

| | |
|---|---|
| FT | Interpolation value |
| I | DO instruction parameter |
| K | Place in table where found |
| T | Point at which interpolation value is needed |
| X | Table containing point at which interpolation occurs |
| Y | Table to be interpolated in |

<u>LAGRA</u>

| | |
|---|---|
| ANS1 | Interpolated value |
| ANS2 | Interpolated derivative value |
| A1, A2, A3 | Differences between input T and tabular values in table TIME |
| B1, B2, B3 | Differences between three consecutive values of T in table TIME |
| C1, C2, C3 | Terms used in evaluation formulas |
| L | DO instruction parameter |
| N | Number of entries in each table |
| NN | N - 1 |
| NSWIT | -1 only derivative, 0 both, 1 only function |

| | |
|---|---|
| T | Point at which interpolated value is to be found |
| TABLE | Table to be interpolated in |
| TIME | Table which contains values of T |

<u>AREA</u>

| | |
|---|---|
| COSET | $\cos \eta$ |
| CTH2 | $\cos \theta_o^2$ |
| IDEL | Number of print steps - 1 |
| SINET | $\sin \eta$ |

<u>DERIZ</u>

| | |
|---|---|
| AST | $a \cdot \sin \theta$ |
| J | Setting of sense switch 5 |
| VV | Value of wind velocity at given z |

<u>OUTPU</u>

| | |
|---|---|
| TAUA | Integrand for AGE integral |

<u>DERPR</u>

| | |
|---|---|
| J | Setting of sense switch 5 |

<u>OUTPR</u>

| | |
|---|---|
| P1 | $\Delta p/p$ |
| S | SINT - 0.5 $\tau_{age}$ $V_E^2(\xi)$ |

| | |
|---|---|
| SINT | $\int V_E(\xi)\, d\xi$ |
| XIO | Linear phase (time) $\xi$ |
| XI1 | Actual phase (time) $\xi_1$ |

NODE

| | |
|---|---|
| A, B, C | Coefficients for RKG starting procedure |
| CK | Subterm in RKG evaluation |
| COMPD | Name of subroutine which calculates integrands |
| COMPE | Name of subroutine accessed before doubling and halving |
| COMPT | Name of subroutine to produce output |
| COMPY | Used in differential equations |
| I,J,K | DO range parameters |
| NN | Number of integrals plus saved quantities |
| NSWHF | Rehalving switch |
| PERR | Maximum error at step |
| R | Subterm in RKG evaluation |
| TEMP | Predictor value of variable |
| TEMPA | Error for $n^{th}$ equation |
| XSAVE | Value for independent variable when step was halved |

GROUN

| | |
|---|---|
| FI | $\phi$ for given ground data |
| I | position in tables at which current data are stored |

| | |
|---|---|
| IC | Step for moving and for looking up in tables |
| IMIN | Position in main table where current table starts |
| ISAVE | Place in tables where values are stored during permutation |
| J | Place in current T-table which has minimum value of time |
| JMAX | Place in tables where current table ends |
| K | Place current value is stored in N-table |
| KERR | Indicates when there is no TVAL in table |
| KI | Next position in table and DO parameter |
| KK | Table number |
| KMAX | Last table used for current curve |
| KN | Starting place of current table |
| LOOPS | Place moved from |
| N | Table of table lengths |
| NMNSW | Switch to indicate first negative $\phi$ for maneuver point |
| NPHSW | Switch to indicate first maneuver point |
| NREC | DO range parameter |
| NRMAX | Number of ground intersections saved |
| P | Table of pressure values at the ground |
| PVAL | Pressure value at time TVAL in $KK^{th}$ table |
| T | Table of time values at the ground |
| TMIN | Minimum value of time for the current table |
| TVAL | Time along curve being plotted |
| X | Table of x-values at the ground |
| XVAL | x-value at time TVAL in $KK^{th}$ table |

| | |
|---|---|
| Y | Table of y-values at the ground |
| YVAL | y-value at time TVAL in $KK^{th}$ table |

INTEP

| | |
|---|---|
| IN | Place in T-table where current table starts |
| INF | Place in T-table where current table ends |
| IS | DO instruction parameter |
| KK | Place in table N which gives position in table T |
| N | Table of table lengths |
| NERR | Set to 1 if no time TVAL in table, otherwise 0 |
| P | Table of pressure values at the ground |
| PANS | Pressure at time TVAL |
| PERCT | Distance between table entries for values |
| T | Table of time values at the ground |
| TVAL | Entry in time table at which interpolation is made |
| X | Table of x-values at the ground |
| XANS | x at time TVAL |
| Y | Table of y-values at the ground |
| YANS | y at time TVAL |

MOVE

| | |
|---|---|
| J | Position in tables moved from |
| K | Position in tables moved to |

RESTA

C COMMON block

INITAL

C COMMON block

TABLE 2.

COMMENTS GENERATED IN PROGRAM

| Comment | Interpretation |
|---|---|
| 1. ALTITUDE VALUES NOT INCREASING | Altitude in tables must increase monotonically; correct data and rerun the problem. |
| 2. TIME IN TABLE NOT INCREASING | Time in maneuver table must increase monotonically; correct data and rerun problem. |
| 3. DELTA PHI IS ZERO WITH PHIMAX NOT ZERO | Maximum $\phi$ must be zero if $\delta\phi$ is zero; correct data and rerun problem. |
| 4. TABLES TOO SHORT | Tables should have at least three entries. Revise data and rerun problem. |
| 5. ABSCISSA VALUE OUTSIDE RANGE | Linear interpolation is attempted outside the range of the table. Execution continues but answers will not be correct. Revise table or the problem statements. |
| 6. AREA ZERO | Area in ray tube diminishes to zero. Current ray calculation is terminated. Age may not be exactly correct. |
| 7. RAY HORIZONTAL | Ray has turned to become within $2^{o}$ of horizontal. Current ray calculation is terminated. |
| 8. NEGATIVE SQUARE ROOT IN AGE | Current ray calculation is terminated. |
| 9. ALTITUDE LARGER THAN 52 000 METERS | Altitude outside range of standard atmosphere program; execution is terminated. |

TABLE 3.

FORMATS FOR INPUT DATA

Card 1 Title card

Card 2 (All input is fixed point, right justified)

| | | | | | | |
|---|---|---|---|---|---|---|
| Cols 1-5 | 6-10 | 11-15 | 16-20 | 21-25 | 26-30 | 31-35 |
| IUNIT | ISTAN | IALT | ITEMP | IRHO | IM | NPHI |
| Cols 36-40 | 41-45 | 46-50 | 51-55 | | | |
| NTAU | NALTS | NWIND | NIM | | | |

Card 3 (All input is floating point F10.3, except last item is E20.7, right justified)

| | | | | |
|---|---|---|---|---|
| Cols 1-10 | 11-20 | 21-30 | 31-40 | 41-60 |
| WS | LA | HG | HO | RE |

Cards 4 to (3 + NPHI × NTAU)(F10.3)

| | | |
|---|---|---|
| Cols 1-10 | 11-20 | 21-30 |
| PHI | L/LA | F |

List L/LA and F for a given $\phi_r$, then proceed to next larger $\phi_r$ and list corresponding L/LA and F .

If loading nonstandard atmosphere, use Cards 5 or 6; otherwise omit.

Cards 5 to (4 + NALTS)(F10.3)

| | | |
|---|---|---|
| Cols 1-10 | 11-20 | 21-30 |
| ALT | TEMP | PRESS |

Cards 6 to (5 + NALTS)(F10.3)

| | | |
|---|---|---|
| Cols 1-10 | 11-20 | 21-30 |
| ALT | TEMP | RHO |

Cards 7 to (6 + NWIND)(F10.3)

| Cols 1-10 | 11-20 | 21-30 |
|---|---|---|
| HW | VVW | ETA |

Cards 8 to (7 + NIM)(F10.3)

If IM = 1

| Cols 1-10 | 11-20 | 21-30 | 31-40 | 41-50 | 51-60 | 61-70 |
|---|---|---|---|---|---|---|
| T | NT | NL | M | GR | PSIR = LAMR | FIA |

If IM = 2

| Cols 1-10 | 11-20 | 21-30 | 31-40 | 51-60 |
|---|---|---|---|---|
| T | M | GR | PSIR = LAMR | FIA |

Card 9 (F10.3)

| Cols 1-10 | 11-20 |
|---|---|
| TINIT | TSTEP |

Card 10 (F10.3)

| Cols 1-10 | 11-20 | 21-30 | 31-40 | 41-50 | 51-60 | 61-70 |
|---|---|---|---|---|---|---|
| ZSTEP | ZPRN | DELFI | FIMAX | PRIN | RF | FF |

NOTES:

1. Altitude means geometric altitude except in input when geopotential option is used.

2. Angles in input and output are in degrees.

3. Output listings are English units. The wind speed in output listings has units ft/sec, whereas in input the units are knots. The unit for pressure is $lb/ft^2$ .

4. Pressure, density, temperature and F-function tables are interpolated linearly, whereas other input tables are interpreted quadratically. Therefore the tables require a minimum of three input cards. The inputs must be selected to avoid wrong quadratic representation between the specified data.

5. The F-function used in the computation is the tabular data for the $\phi$ nearest the $\phi$ being computed.

6. For a steady-state flight, a minimum of three maneuver cards must be used with arbitrary time T increasing from the first card to the last. To avoid duplicate solutions, select TSTEP so that TINIT + TSTEP is larger than the latest time T .

7. If both $\Delta\phi$ and $\phi_{max}$ are input as zero, only the $\phi = 0$ ray is computed. If $\Delta\phi$ is input as zero and $\phi_{max}$ is nonzero, an error statement is given and no solution is made. If $\Delta\phi$ is input as a negative number, only the rays for $\phi = 0$ and negative $\phi$'s are computed.

TABLE 4.

INPUT DEFINITIONS

| | | |
|---|---|---|
| IUNIT: | -1--ENGLISH<br>0--METRIC<br>+1--SPECIAL | Input units; outputs are English units |
| ISTAN: | 1--1962 US Standard Atmosphere<br>2--Tabular atmosphere | |
| IALT: | 1--Geopotential altitude<br>2--Geometric altitude | input for tabular atmosphere and wind data |
| ITEMP: | 1--Fahrenheit<br>2--Centigrade<br>3--Rankine | |
| IRHO : | 1--Altitude, temperature, pressure table<br>2--Altitude, temperature, density table | |
| IM : | 1--Maneuver, option 1<br>2--Maneuver option 2 | |
| NPHI : | Number of PHI's ($\phi$) in F-function table | |
| NTAU : | Number of length parameters L/LA in F-function table | |
| NALTS: | Number of entries in IRHO table (Cards 5 or 6) | |
| NWIND: | Number of entries in wind table (Cards 7) | |
| NIM : | Number of entries in maneuver table (Cards 8) | |
| WS : | Wing loading, W/S | |
| LA : | Length of aircraft | |
| HG : | Altitude of ground above sea level | |
| HO : | Altitude of aircraft above sea level | |
| RE : | Radius of earth (e.g., $2.089007 \times 10^7$ ft or $6.367293 \times 10^6$ m) | |
| PHI : | Ray azimuth angle relative to aircraft z-axis, $\phi_r$ | |

| | | |
|---|---|---|
| FI : | Ray azimuth angle relative to vertical plane, $\phi$ | |
| L/LA : | Stations at which F-function is specified | |
| F : | Aircraft signature function | |
| ALT :<br>TEMP :<br>PRESS:<br>RHO : | Atmospheric temperature and either pressure or density as functions of altitude above sea level. Altitude (ALT) may be geopotential or geometric. | |
| HW :<br>VWW :<br>ETA : | Wind speed and direction from which wind is coming as functions of altitude (HW) above sea level. Direction is the heading angle measured eastward from north. | |
| T : | Aircraft flight time | |
| NT : | Axial load factor | |
| NL : | Lift (normal) load factor | |
| M : | Mach number | |
| GR : | Flight path angle above horizontal | relative to the atmosphere, including wind |
| PSIR : | Heading angle measured clockwise from north (eastward) | |
| FIA : | Bank angle of aircraft, $\phi_a$ | |
| TINIT: | Initial time at which first ray tracing is to be calculated (must correspond to any T input) | |
| TSTEP: | Time interval along aircraft flight path at which ray calculations will be initiated | |
| ZSTEP: | Initial ray-area integration interval of time (DELZ1) | |
| ZPRN : | Altitude intervals at which ray data are to be listed (DELPT) | |
| DELFI: | Increments of FI (ray azimuth angles) at which calculations are desired | |
| FIMAX: | Maximum FI desired | |
| PRIN : | A selected altitude at which ray data are desired | |
| RF : | Ground reflection factor (normally 1.8 to 2.0) | |
| FF : | F-function factor to multiply input parameter $F_i$ | |

TABLE 5.

INPUT UNITS AND CONVERSION FACTORS

| | OPTION (-1) English | OPTION (0) Metric | OPTION (+1) Special |
|---|---|---|---|
| Atmospheric temperature, T : | $^{o}F$ or $^{o}C$ or $^{o}R$ | $^{o}F$ or $^{o}C$ or $^{o}R$ | $^{o}F$ or $^{o}C$ or $^{o}R$ |
| Atmospheric pressure, p: | $lb/ft^2$ | $N/m^2$ | mbars |
| Atmospheric density, $\rho$ : | $slug/ft^3$ | $kg/m^3$ | $kg/m^3$ |
| Wind speed, $V_W$ or u : | knots | m/sec | knots |
| Wind direction, $\eta$ : | deg | deg | deg |
| Length or altitude : | ft | m | ft |
| Wing loading, W/S : | $lb/ft^2$ | $kg/m^2$ | $lb/ft^2$ |

CONVERSION FACTORS

| | | |
|---|---|---|
| ft to m | : | multiply by 0.3048 |
| $lb/ft^2$ to mbars | : | 0.47880258 |
| $N/m^2$ | : | 47.880258 |
| $kg/m^2$ | : | 4.88242 |
| $slugs/ft^3$ to $kg/m^3$ | : | 515.379 |
| knots to ft/sec | : | 1.68781 |
| m/sec | : | 0.514444 |
| $^{o}F$ to $^{o}C$ | : | $^{o}C = (5/9)(^{o}F - 32)$ |
| $^{o}F$ to $^{o}R$ | : | $^{o}R = {}^{o}F + 459.67$ |

## APPENDIX A

## PROGRAM LISTING

```
      PROGRAM SONIC
      EXTERNAL   DERM2,OUTM2,PASS
      REAL M,LAMDO  ,MM(25),LAMR(25),NT(25),NL(25),LA,MU,MUDOT
      DIMENSION PHI(20),TAU(20),F(20,20),FIA(25),
     1 ALT(25),TEMP(25),PRESS(25),HW(25),VVW(25),ETA(25),TT(25),GR(25)
      COMMON NNODE,HNODE,TNODE,ACC,NODUM,BYHAL  ,NODUB,ENDNO,ENDVL,NSAVE,
     5FLAG,TIND,VAR(14,8)
      COMMON IUNIT,ISTAN,IALT,ITEMP,IRHO,IM,NPHI,NTAU,NALTS,NWIND,NIM
     1,WS,HG,HO,RE,PHI,TAU,F,FIA,ALT,TEMP,PRESS,RF, HW,VVW,ETA,MU,MUDOT,
     2TT,MM,GR,LAMR,NT,NL,LA,TINIT,GDOT,DTPRN,DELZ1,DELPT,FIMAX,DELFI,PR
     3IN,MTIME,KTIME,M,V,VWX,VWY,GAMR,PSIDT,RHOO,LAMDO,TPRIN,TIME,ZINIT,
     5XM,YM,A,BETA,LUNIT,DADZ,VW,EETA,VWDZ,ETADZ, MUNIT,FI,FA,DEL,TLAS
C
C    EXECUTIVE PROGRAM FOR THE SONIC BOOM CALCULATION
      CALL SSWTCH(1,NNODE)
      GO TO (4,1),NNODE
C    THE REAL BEGINNING OF A DATA CASE
    1 CALL INPU
    2 FI=0.
      DEL = DELFI
C        MAIN PROGRAM FOR MANEUVERS
   80 IF(TPRIN-TT(NIM)) 70,70,20
   70 IF(ABS(TPRIN-TT(NIM))-.01) 20,20,10
   20 NNODE = KTIME+1
      WRITE(MUNIT)        FI,FI,FI,FI,FI
      NODUM = MUNIT
      NODUB = LUNIT
      REWIND MUNIT
      CALL GROUN
      GO TO 1
   10 TIND = TPRIN
      TLAS   = -9999.
      VAR(1,1) = XM
      VAR(1,2) = YM
      VAR(1,3) = ZINIT
      GO TO (30,40),MTIME
   30 MTIME = 2
      ENDNO = 0.
      CALL DERM2
      CALL OUTM2
C     CALL RAY TRACING THE FIRST TIME
      GO TO 3
   40 TPRIN = TPRIN + DTPRN
      NSAVE = 0
      ENDVL = TPRIN
      NNODE = 3
      FLAG = 0.
      ACC = 1.
      TNODE = 1.
      NODUM = 0
      NODUB = 0
      BYHAL = .5
      ENDNO = 10.
   50 CALL NODE ( DERM2,DERM2,OUTM2,PASS)
    3 CALL AREA
      GO TO 2
    4 CALL INITAL
      GO TO 3
      END
```

```
      SUBROUTINE INPU
      REAL M,LAMDO ,MM(25),LAMR(25),NT(25),NL(25),LA,MU,MUDOT
      DIMENSION PHI(20),TAU(20),F(20,20),FIA(25),
     1 ALT(25),TEMP(25),PRESS(25),RHO(25),HW(25),
     2 VVW(25),ETA(25),TT(25),GR(25)
      COMMON N,H,T,ACCUR,NODUM,REDUC,NODUB,ENDNO,ENDVL,NSAVE,FLAG,Z,VAR
     1(14,8)
      COMMON IUNIT,ISTAN,IALT,ITEMP,IRHO,IM,NPHI,NTAU,NALTS,NWIND,NIM
     1,WS,HG,HO,RE,PHI,TAU,F,FIA,ALT,TEMP,PRESS,RF, HW,VVW,ETA,MU,MUDOT,
     2TT,MM,GR,LAMR,NT,NL,LA,TINIT,GDOT,DTPRN,DELZ1,DELPT,FIMAX,DELFI,PR
     3IN,MTIME,KTIME,M,V,VWX,VWY,GAMR,PSIDT,RHOO,LAMDO,TPRIN,TIME,ZINIT,
     5XM,YM,A,BETA,LUNIT,DADZ,VW,EETA,VWDZ,ETADZ,MUNIT
      EQUIVALENCE(PRESS(1),RHO(1))
C
C
C   IUNIT =     NEGATIVE NUMBER FOR INPUT VALUES IN ENGLISH SYSTEM
C   IUNIT = 0                      FOR INPUT VALUES IN METRIC SYSTEM
C   IUNIT = POSITIVE NUMBER FOR INPUT VALUES IN ESSA
C
C   ISTAN = 1 FOR STANDARD  ATMOSPHERIC DATA
C   ISTAN = 2 FOR TABULAR ATMOSPHERIC INPUT
C
C   IALT = 1  FOR ALTITUDE IN GEOPOTENTIAL UNITS
C   IALT = 2 FOR ALTITUDE IN GEOMETRIC UNITS
C
C   ITEMP = 1 FOR TEMPERATURE INPUT IN DEGREES FARENHEIGHT
C   ITEMP = 2 FOR TEMPERATURE IN DEGREES CENTIGRADE
C     ITEMP=  3 FOR TEMPERATURE IN DEGREES RANKINE
C
C   IRHO =1 FOR ALTITUDE,TEMPERATURE, PRESSURE TABLE
C   IRHO = 2 FOR ALTITUDE,TEMPERATURE,DENSITY,TABLE
C
C   IM = 1 FOR MANEUVER OPTION 1,TABLES OF LOADFACTORS, GAM,PSI AND BANK
C   IM   IM = 2 FOR MANEUVER TWO, FOR MACH, GAMMA AND PSI TABLES
C
C NIM IS THE NUMBER OF ENTRIES IN THE MANUEVER INPUT TABLES
C
C NPHI IS THE NUMBER OF PHI'S IN THE F FUNCTION TABLE
C NTAU IS THE NUMBER OF TAU'S IN THE F FUNCTION TABLE
C
C  NALTS IS THE NUMBER OF ENTRIES IN THE ALTITUDE,TEMPERATURE,
C  PRESSURE/DENSITY TABLE
C
C NWIND IS THE NUMBER OF ENTRIES IN THE WIND VELOCITY TABLE
C
C  RE IS THE RADIUS OF THE EARTH
C
C
C*****READ CONTROL CARD
      KUNIT = 5
      LUNIT = 6
      MUNIT = 8
      READ (KUNIT,4)
      READ(KUNIT,1)
     1          IUNIT,ISTAN,IALT,ITEMP,IRHO,IM,NPHI,NTAU,NALTS,NWIND,
     2NIM
    1 FORMAT(11(2X,I3))
C*****PRINT CONTROL VALUES
      WRITE(LUNIT,8)
      WRITE (LUNIT,4)
      WRITE (LUNIT,2)
```

```
      WRITE(LUNIT,9)
     1              (ALT(I),TEMP(I),RHO(I),I=1,NALTS)
 202  WRITE(LUNIT,15)
      WRITE(LUNIT,16)
     1              (HW(I),VVW(I),ETA(I),I=1,NWIND)
      GO TO (70,60),IM
  60  WRITE(LUNIT,61)
      WRITE(LUNIT,62)
     1              (TT(I),MM(I),GR(I),LAMR(I),FIA(I),I=1,NIM)
      GO TO 900
  70  WRITE(LUNIT, 71)
      WRITE(LUNIT,72)
     1              (TT(I),NT(I),NL(I),MM(I),GR(I),LAMR(I),FIA(I),I=1,NIM)
 900  WRITE(LUNIT,91) TINIT,DTPRN
      WRITE(LUNIT,92) DELZ1,DELPT,PRIN,DELFI,FIMAX,RF,FF
      IF(DELFI) 160, 155,160
 155  IF(FIMAX)  158,156,158
 158  WRITE(LUNIT,159)
 159  FORMAT(20X, 39HDELTA PHI IS ZERO WITH PHI MAX NOT ZERO)
      CALL EXIT
 250   WRITE (LUNIT,251)
 251  FORMAT(//20X30HATLITUDE VALUES NOT INCREASING      )
      CALL EXIT
 252  WRITE (LUNIT,253)
 253  FORMAT(//20X,28HTIME IN TABLE NOT INCREASING     )
      CALL EXIT
 156  DELFI =-.00005
C
C
C CONVERSIONS
C
C
C CONVERSION TO GEOMETRIC UNITS IF ALTITUDE IN GEOPOTENTIAL UNITS
C
 160  GO TO (200,204),ISTAN
 204  IF(NALTS-2) 150,150,205
 205  GO TO(201,203),IALT
 201  DO 181 I = 1,NALTS
 181  ALT(I) = (ALT(I)*RE)/(RE-ALT(I))
C
C CONVERSION TO Z
C
 203  DO 183 I=1,NALTS
 183  ALT(I)=ALT(I)-HG
C
C          CHECK FOR TABLES TO HAVE INCREASING VALUES
      DO 206 I = 2,NALTS
      IF(ALT(I) - ALT(I-1)) 250,250,206
 206  CONTINUE
 200  GO TO (1820,1821),IALT
1820  DO 182 I=1,NWIND
 182  HW(I)=(HW(I)*RE)/(RE-HW(I))
1821  DO 184 I=1,NWIND
 184  HW(I) = HW(I) -HG
      DO 207 I = 2,NWIND
      IF(HW(I)- HW(I-1)) 250,250,207
 207  CONTINUE
      DO 208 I = 2,NIM
      IF(TT(I)-TT(I-1)) 252,252,208
 208  CONTINUE
C
C CONVERSION TO ENGLISH SYSTEM IF NECESSARY
```

```
     1            IUNIT,ISTAN,IALT,ITEMP,IRHO,IM,NPHI,NTAU,NALTS,NWIND,NIM
      IF(NWIND-2) 150,150,151
  150 WRITE(LUNIT,152)
      CALL EXIT
  152 FORMAT(20X ,20HTABLES TOO SHORT                    )
  151 IF(NIM  -2) 150,150,154
C READ                          WING LOADING,LENGTH OF AIRPLANE, ALTITUDE
C OF THE GROUND, ALTITUDE OF THE AIRCRAFT, AND RADIUS OF THE EARTH
  154 READ(KUNIT,3)
     1            WS,LA,HG,HO,RE
    3 FORMAT(4F10.3,E20.7)
    8 FORMAT(1H1)
    4 FORMAT(80H
     1                                                                  )
C READ AIRCRAFT SIGNATURE, F FUNCTION
      READ(KUNIT,5)
     1           ((PHI(J),TAU(I),F(I,J),I=1,NTAU),J=1,NPHI)
      GO TO(103,22),ISTAN
C READ ALTITUDE,TEMPERATURE ,PRESSURE/DENSITY TABLE
   22 GO  TO (101,102),IRHO
  101 READ(KUNIT,5)
     1            (ALT(I),TEMP(I),PRESS(I), I=1,NALTS)
      GO TO 103
  102 READ(KUNIT,5)
     1            (ALT(I),TEMP(I),RHO(I),I=1,NALTS)
C READ WIND VELOCITY TABLE
C HW IS THE ALTITUDE, VVW IS THE WIND SPEED, AND ETA IS DEGREES FROM
C NORTH
  103 READ(KUNIT,5)
     1           (HW(I),VVW(I),ETA(I),I=1,NWIND)
      CALL ANGLE(ETA,NWIND)
      IF(IM-1) 7,7,6
    6 READ(KUNIT,55)
     1            (TT(I),MM(I),GR(I),LAMR(I),FIA(I),I=1,NIM)
      GO TO 90
    7 READ(KUNIT,57)
     1            (TT(I),NT(I),NL(I),MM(I),GR(I),LAMR(I),FIA(I),I=1,NIM)
      CALL ANGLE(FIA,NIM)
   90 READ(KUNIT,5) TINIT,DTPRN
      CALL ANGLE(GR,NIM)
      CALL ANGLE(LAMR,NIM)
      READ(KUNIT,58)DELZ1,DELPT,DELFI,FIMAX,PRIN,RF,FF
   58 FORMAT(7F10.3)
      FIMAX = ABS(FIMAX)
C
C
C*****PRINT OUT ALL INPUT DATA
C
C
      WRITE(LUNIT,141) WS,LA
      WRITE(LUNIT,14) HG,HO,RE
      WRITE(LUNIT,13)
      WRITE(LUNIT,12)
     1              ((PHI(J),TAU(I),F(I,J),I=1,NTAU),J=1,NPHI)
      GO TO (202,222),ISTAN
  222 GO TO(111,112),IRHO
  111 WRITE(LUNIT,10)
      WRITE(LUNIT,9)
     1            (ALT(I),TEMP(I),PRESS(I),I=1,NALTS)

      GO TO 202
  112 WRITE(LUNIT,11)
```

```
C
      IF(IUNIT)3030,301,338
C
C CONVERSION FOR METRIC TO ENGLISH
  301 HG =HG/.3048
      HO=HO/.3048
      LA=LA/.3048
      WS = WS/4.88242
      RE=RE/.3048
      DO 381 I=1,NWIND
      HW(I) = HW(I)/.3048
  381 VVW(I)=(VVW(I)/.3048)
      GO TO(405,406),IRHO
  405 DO 483 I = 1,NALTS
      ALT(I)=ALT(I)/.3048
  483 PRESS(I)=PRESS(I)/47.88026
      GO TO 333
  406 DO 484 I=1,NALTS
      ALT(I)=ALT(I)/.3048
  484 RHO(I)=RHO(I)/515.379
      GO TO 333
C
C CONVERSION FROM ESSA
C
  338 GO TO (3030,3338),ISTAN
 3338 GO TO(601,602),IRHO
  601 DO 681 I=1,NALTS
 681  PRESS(I)=PRESS(I)/ .4788026
      GO TO 3030
  602 DO 682 I=1,NALTS
 682  RHO(I)=RHO(I)/515.379
C  CONVERT WIND SPEED FROM KNOTS TO FEET PER SECOND
 3030 DO 782 I=1,NWIND
  782 VVW(I) =VVW(I)*1.688944
C        CONVERT TEMPERATURE TO RANKINE
  333 GO TO (303,3031),ISTAN
3031  GO TO (901,902,303),ITEMP
 901  DO 981 I=1,NALTS
 981  TEMP(I)=TEMP(I)+459.7
      GO TO 303
902   DO 982 I=1,NALTS
982   TEMP(I)=(9./5.)*(TEMP(I)+273.15)
C CONVERT ALL DEGREES TO RADIANS
  303 DO 781 I=1,NPHI
  781 PHI(I) =PHI(I)*.1745329E-01
      DO 783 I=1,NWIND
  783 ETA(I) = ETA(I)*.1745329E-01
  784 DO 786 I=1,NIM
      GR(I) = GR(I)*.1745329E-01
  786 LAMR(I)=LAMR(I)*.1745329E-01
  787 DO 788 I =1,NIM
  788 FIA(I) =FIA(I)*.1745329E-01
    2 FORMAT(/10X,6HIUNIT=, I3,  10X,  6HISTAN=,I3,10X,  6HIALT= I3,10X,
     1  6HITEMP=,I3,10X,  5HIRHO=,  I3,  10X,3HIM=, I3,//
     2  10X,6HNPHI= ,I3,10X,6HNTAU= ,I3,10X,6HNALTS=,I3,10X,6HNWIND=,I3,
     3  10X, 5HNIM= ,I3 ///)
    5 FORMAT(3F10.3)
    9 FORMAT(17X,F7.0,22X,F5.0,15X,E20.7)
   10 FORMAT(//    ,16X,8HALTITUDE,19X,11HTEMPERATURE,21X,8HPRESSURE)
   11 FORMAT(//    ,16X,8HALTITUDE,19X,11HTEMPERATURE,22X,7HDENSITY)
   12 FORMAT(20X,F5.2,10X,F5.2,10X,F10.5)
```

```
 13 FORMAT(21X,3HPHI,12X,4HL/LA  ,16X,1HF)
 14 FORMAT(10X,*HG=*,F7.0,10X,*HO=*,F7.0,10X,*RE=*E20.7///)
 15 FORMAT(//     19X,8HALTITUDE,8X,10HWIND SPEED,5X,9HDIRECTION)
 16 FORMAT(20X,F7.0,8X,F8.2,9X,F6.1)
 55 FORMAT(5F10.3)
 57 FORMAT(7F10.3)
 61 FORMAT(///23X,4HTIME,17X,7HMACH NO,14X,5HGAMMA,14X,4H PSI,10X,10HB
   1ANK ANGLE)
 62 FORMAT(20X,F10.3,10X,F10.3,10X,F10.3,10X,F10.3,10X,F10.3)
 71 FORMAT(//     13X,4HTIME,15X,*NT*,15X*NL*10X,*MACH NO*12X*GAMMA*11X
   1,*PSI*10X,*BANK ANGLE*)
 72 FORMAT(7F17.3)
 91 FORMAT(//1X,13HINITIAL TIME=,F10.3,13X,14HDELTA T PRINT=,F14.5//)
 92 FORMAT(1X,8HDELTA Z=,F14.5,14X,15HPRINT INTERVAL=,F10.3,//,1X
   2 16HPRINT OUT POINT=,F10.3,10X,5HDPHI=,F10.3,10X,12HMAXIMUM PHI=,
   3  F10.3//1X,19HREFLECTION FACTOR =, F10.3, 7X,10HF FACTOR =,F10.3)
141 FORMAT(10X,13HWING LOADING=,F10.3,10X,16HAIRPLANE LENGTH=,F10.3
   1///)
785 TPRIN = TINIT
    DO 789 J = 1,NPHI
    DO 789 I = 1,NTAU
789 F(I,J) = FF*F(I,J)
    ZINIT = HO-HG
    XM = 0.
    YM = 0.
    MTIME = 1
    KTIME = 0
    REWIND MUNIT
    RETURN
    END
```

```
      SUBROUTINE ATMO (Z,TH,P,D,VS)
      REAL K1(5),K23(5)
      REAL M,LAMDO ,MM(25),LAMR(25),NT(25),NL(25),LA,MU,MUDOT
      DIMENSION HGLTB(6),PB(5),RHOB(5),TB(5)
      DIMENSION PHI(20),TAU(20),F(20,20),FIA(25),RHO(25),
     1 ALT(25),TEMP(25),PRESS(25),HW(25),VVW(25),ETA(25),TT(25),GR(25)
      COMMON NNODE,HNODE,TNODE,ACC,NODUM,BYHAL ,NODUB,ENDNO,ENDVL,NSAVE,
     5FLAG,TIND,VAR(14,8)
      COMMON IUNIT,ISTAN,IALT,ITEMP,IRHO,IM,NPHI,NTAU,NALTS,NWIND,NIM
     1,WS,HG,HO,RE,PHI,TAU,F,FIA,ALT,TEMP,PRESS,RF, HW,VVW,ETA,MU,MUDOT,
     2TT,MM,GR,LAMR,NT,NL,LA,TINIT,GDOT,DTPRN,DELZ1,DELPT,FIMAX,DELFI,PR
     3IN,MTIME,KTIME,M,V,VWX,VWY,GAMR,PSIDT,RHOO,LAMDO,TPRIN,TIME,ZINIT,
     5XM,YM,A,BETA,LUNIT,DADZ,VW,EETA,VWDZ,ETADZ,MUNIT
      EQUIVALENCE(PRESS(1),RHO(1))
C
C H IS THE HEIGHT IN METERS FOR WHICH A CORRESPONDING TEMPERATURE,
C PRESSURE AND DENSITY WILL BE COMPUTED
C
C HGLTB,K1,K23,TB,PB,RHOB   ARE STANDARD ATMOSPHERE DATA ARRAYS
C
C TH,P,D ,VS,   ARE THE TEMPERATURE,PRESSURE,DENSITY,AND SPEED OF SOUND
C TO BE COMPUTED AND RETURNED TO THE CALLING PROGRAM
C
C
C
      HGLTB(1) =0.
      K1(1)    =-.225571E-04
      K23(1)   =-5.255894
      TB(1)    =518.670
      PB(1)    =2116.21695
      RHOB(1)  =2.37692E-03
      HGLTB(2) =11000.
      K1(2)    =0.
      K23(2)   =.157689E-03
      TB(2)    =389.970
      PB(2)    =472.68
      RHOB(2)  =7.06126E-4
      HGLTB(3) =20000.
      K1(3)    =.461574E-05
      K23(3)   =34.163426
      TB(3)    =389.970
      PB(3)    =114.346
      RHOB(3)  =.170817E-3
      HGLTB(4) =32000.
      K1(4)    =.12245E-04
      K23(4) = 12.20122
      TB(4)    =411.57
      PB(4)    =18.129
      RHOB(4)  =2.56609E-05
      HGLTB(5) =47000.
      K1(5)    =0.
      K23(5) =.12622733E-03
      TB(5)    =487.17
      PB(5)    =2.3163
      RHOB(5)  =2.76983E-6
      HGLTB(6) =52000.
C TO FIND LAYER NUMBER
C
      H=Z+HG
      HGP=0.3048*H/(1.0+0.3048*H/6356766.0)
      IF(HGP-53000.) 1,5,5
```

```
    1 DO 20 I=1,5
      IF(HGLTB(I)-HGP)2,3,7
    2 J=I
   20 CONTINUE
      GO TO 7
    3 J=I
      GO TO 7
    5 WRITE(LUNIT,6)
    6 FORMAT(//20X33HALTITUDE LARGER THAN 52000 METERS)
      CALL EXIT
C
C TO COMPUTE TEMPERATURE,DENSITY,PRESSURE AND SPEED OF SOUND
C
    7 TEM1=HGP-HGLTB(J)
      TH=TB(J)
      THDZ = TH*K1(J)
      IF(K1(J))103,104,103
  103 TEM1=1.0+K1(J)*TEM1
      TH=TH*TEM1
      TEM2=TEM1**(-1.0-K23(J))
      D=   RHOB(J)*TEM2
      P     =PB(J)*TEM1*TEM2
      GO TO 105
00104 TEM1=EXP(-K23(J)*TEM1)
      P =    PB(J)*TEM1
      D   =RHOB(J)*TEM1
  105 VS=49.02057*TH**0.5
      DADZ = (1201.5081*THDZ)/VS
      RETURN
      END
```

```
      SUBROUTINE ANGLE(ARRAY,MAX)
      DIMENSION ARRAY(10)
      MAXS = MAX - 1
      DO 4  I = 1,MAXS
    9 IF(ABS(ARRAY(I+1)-ARRAY(I))-180.) 4,4,8
    8 IF(ARRAY(I+1)-ARRAY(I)) 10,4,11
   10 ARRAY(I+1) = ARRAY(I+1) +360.
      GO TO 9
   11 ARRAY(I+1) = ARRAY(I+1) -360.
      GO TO 9
    4 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE DERM2
      REAL M,LAMDO ,MM(25),LAMR(25),NT(25),NL(25),LA,MU,MUDOT
      DIMENSION PHI(20),TAU(20),F(20,20),FIA(25),RHO(25),
     1 ALT(25),TEMP(25),PRESS(25),HW(25),VVW(25),ETA(25),TT(25),GR(25)
      COMMON NNODE,HNODE,TNODE,ACC,NODUM,BYHAL ,NODUB,ENDNO,ENDVL,NSAVE,
     5FLAG,TIND,VAR(14,8)
      COMMON IUNIT,ISTAN,IALT,ITEMP,IRHO,IM,NPHI,NTAU,NALTS,NWIND,NIM
     1,WS,HG,HO,RE,PHI,TAU,F,FIA,ALT,TEMP,PRESS,RF, HW,VVW,ETA,MU,MUDOT,
     2TT,MM,GR,LAMR,NT,NL,LA,TINIT,GDOT,DTPRN,DELZ1,DELPT,FIMAX,DELFI,PR
     3IN,MTIME,KTIME,M,V,VWX,VWY,GAMR,PSIDT,RHOO,LAMDO,TPRIN,TIME,ZINIT,
     5XM,YM,A,BETA,LUNIT,DADZ,VW,EETA,VWDZ,ETADZ, MUNIT,FI,FA,DEL,TLAS
      EQUIVALENCE(PRESS(1),RHO(1))
    3 IF (TIND-TLAS ) 5,60,5
    5 CALL LAGRA(TIND,NIM,TT,GR,GAMR,BLANK,1)
      CALL LAGRA(TIND,NIM,TT,LAMR,LAMDO,BLANK,1)
      CALL LAGRA(TIND,NIM,TT,MM,M,BLANK,1)
      TLAS = TIND
      SINLA = SIN(LAMDO)
      COSLA = COS(LAMDO)
      COSGA = COS(GAMR)
   60 CALL LAGRA(VAR(1,3),NWIND,HW,VVW,VW ,BLANK,1)
      CALL LAGRA(VAR(1,3),NWIND,HW,ETA,EETA,BLANK,1)
      GO TO (40,50),ISTAN
   40 CALL ATMO (VAR(1,3),TH,P,D,A)
      GO TO 55
   50 CALL INPOL(ALT,TEMP,VAR(1,3),TM)
      A = 49.020576*SQRT(TM)
   55 V  = A*M
      VWX = VW*SIN(EETA)
      VWY = VW *COS(EETA)
      VAR(8,1) = V *COSGA*SINLA-VWX
      VAR(8,2) = V *COSGA*COSLA-VWY
      VAR(8,3) = V *SIN(GAMR)
      CALL SSWTCH(5,J)
      GO TO (100,200),J
  100 WRITE (LUNIT,155) TIND,VAR(1,1),VAR(1,2),VAR(1,3)
  155 FORMAT(* TIME = * E15.8 ,5X,*X =*,E15.8,5X,*Y =*,E15.8,5X,*Z =*E15
     1.8)
  200 RETURN
      END
```

```
      SUBROUTINE OUTM2
      REAL M,LAMDO ,MM(25),LAMR(25),NT(25),NL(25),LA
     1,MSQ,MUDOT,MU,NTZ,NLZ ,MDOT
      DIMENSION PHI(20),TAU(20),F(20,20),FIA(25),RHO(25),
     1 ALT(25),TEMP(25),PRESS(25),HW(25),VVW(25),ETA(25),TT(25),GR(25)
      COMMON NNODE,HNODE,TNODE,ACC,NODUM,BYHAL ,NODUB,ENDNO,ENDVL,NSAVE,
     5FLAG,TIND,VAR(14,8)
      COMMON IUNIT,ISTAN,IALT,ITEMP,IRHO,IM,NPHI,NTAU,NALTS,NWIND,NIM
     1,WS,HG,HO,RE,PHI,TAU,F,FIA,ALT,TEMP,PRESS,RF, HW,VVW,ETA,MU,MUDOT,
     2TT,MM,GR,LAMR,NT,NL,LA,TINIT,GDOT,DTPRN,DELZ1,DELPT,FIMAX,DELFI,PR
     3IN,MTIME,KTIME,M,V,VWX,VWY,GAMR,PSIDT,RHOO,LAMDO,TPRIN,TIME,ZINIT,
     5XM,YM,A,BETA,LUNIT,DADZ,VW,EETA,VWDZ,ETADZ, MUNIT,FI,FA,DEL,TLAS
      EQUIVALENCE(PRESS(1),RHO(1))
      IF(ENDNO) 20,20,100
   20 FLAG = -1.
      HH =  VAR(1,3) +HG
      GO TO (11,13),IM
   11 WRITE(LUNIT,7)
      G = RE/(RE+HH)
      G =   32.25724*G*G
      GO TO 12
   13 WRITE(LUNIT,1)
   10 CALL LAGRA(TIND,NIM,TT,GR,BLANK,GDOT,-1)
      CALL LAGRA(TIND,NIM,TT,LAMR,BLANK,PSIDT,-1)
      CALL LAGRA(TIND,NIM,TT,MM,M,MDOT,-1)
   12 WRITE (LUNIT,2) TIND,HH
      MSQ = M*M
      BETA = SQRT(MSQ-1.)
      MU = ATAN(1./BETA)
      G1 = 57.29578*GAMR
      G2 = 57.29578*LAMDO
      G3 = 57.29578*MU
      WRITE (LUNIT,3)VAR(1,1),VAR(1,2),VAR(1,3),G1,G2,G3
      GO TO (50,55), ISTAN
   50 CALL ATMO (VAR(1,3),TH,P,RHOO,A)
      GO TO 60
   55 GO TO (70,71),IRHO
   71 CALL INPOL(ALT,RHO ,VAR(1,3),RHOO)
      GO TO 72
   70 CALL INPOL(ALT,PRESS,VAR(1,3),P)
      CALL INPOL(ALT,TEMP,VAR(1,3),TM)
      RHOO = P/(1716.*TM)
   72 CALL LAGRA(VAR(1,3),NALTS,ALT,TEMP,BLANK,THDZ,-1)
      DADZ = (1201.5081*THDZ)/A
   60 Q = .5*RHOO*V*V
      COSMU  = COS(MU)
      SINGA = SIN(GAMR)
      COSGA = COS(GAMR)
      CALL LAGRA(VAR(1,3),NWIND,HW,VVW,BLANK,VWDZ,-1)
      CALL LAGRA(VAR(1,3),NWIND,HW, ETA,BLANK,ETADZ,-1)
      CALL LAGRA(TIND,NIM,TT,FIA,FA,BLANK,1)
      GO TO (61,62),IM
   62 VDOT = A*MDOT+(MSQ*DADZ*SINGA)*A
      C1 =  (WS*SINGA)/Q
      CL = ( WS*COSGA)/Q
      GO TO 63
   61 CALL LAGRA(TIND,NIM,TT,NL,NLZ,BLANK,1)
      CALL LAGRA(TIND,NIM,TT,NT,NTZ,BLANK,1)
      COSDI = COS(LAMDO-EETA)
      SINDI = SIN(LAMDO-EETA)
      PSIDT = (G*SIN(FA)* NLZ        -VAR(8,3)*(VWDZ*SINDI-ETADZ*VW*COSDI
```

```
     1))/(V*COSGA)
      TERM1 = VAR(8,3)*(VWDZ*COSDI+VW*SINDI*ETADZ)
      VDOT = G*(NTZ-SINGA)+COSGA*TERM1
      GDOT = (G*(COS(FA)*NLZ-COSGA)-SINGA*TERM1)/V
      CL = (NLZ*WS)/Q
      C1 = (NTZ*WS)/Q
   63 G1 = 57.29578*GDOT
      MUDOT = (DADZ*SINGA)/COSMU - (VDOT)/(A*COSMU*MSQ)
      G2 = 57.29578*PSIDT
      G3 = 57.29578*MUDOT
      VG = SQRT(VAR(8,1)*VAR(8,1) + VAR(8,2)*VAR(8,2)+VAR(8,3)*VAR(8,3))
      WRITE (LUNIT,4) VAR(8,1),VAR(8,2),VAR(8,3),G1,G2,G3
      WRITE (LUNIT,5) C1,CL,M,Q,V,VDOT
      WRITE (LUNIT,6) VWX,VWY,A,VG
      XM = VAR(1,1)
      YM = VAR(1,2)
      ZINIT = VAR(1,3)
      TIME = TIND
  100 RETURN
    1 FORMAT (1H1 30X *MANEUVER DATA - OPTION TWO*)
    2 FORMAT(*0TIME  =* F11.1,*  H      =*F11.1)
    3 FORMAT(*0XG    =* F11.1,*  YG     =*F11.1,*  ZG     =*F11.1,*  GAMMA
     1  =* F6.2,*  PSI    =* F6.2,*  MU     =*F6.2)
    4 FORMAT(*0XGDOT =* F11.1,*  YGDOT =*F11.1,*  ZGDOT =*F11.1,*  GAMDO
     1T =* F6.2,*  PSIDOT =* F6.2,*  MUDOT =*F10.4)
    5 FORMAT(*0CT-CD =* F11.6,*  CL     =*F11.6,*  M      =*F11.2,*  Q
     1=*  F8.1, *  V     =*F8.1,*  VDOT  =*F12.2)
    6 FORMAT(*0UX    =* F11.4,*  UY     =*F11.4,*  A      =*F11.2,18X,*VG
     1  =* F8.1)
    7 FORMAT (1H1 30X *MANEUVER DATA - OPTION ONE*)
      END
```

```
      SUBROUTINE INPOL(X,Y,T,FT)
C
C   GENERAL INTERPOLATION SUBROUTINE
C
C      X IS THE ABSICISSA ARRAY
C      Y IS THE ORDINATE ARRAY
C      T IS THE ABSICISSA VALUE FOR WHICH A
C      CORRESPONDING ORDINATE VALUE ,FT, WILL BE FOUND
C
C
      DIMENSION X(25),Y(25)
      IF(X(1)-T)11,11,12
   11 DO 1 I=1,25
      IF(X(I)-T)1,2,3
    1 CONTINUE
   12 WRITE(3,10)
   10 FORMAT(1X,29HABSICISSA VALUE OUTSIDE RANGE)
      GO TO 4
    2 FT=Y(I)
      GO TO 4
    3 K=I-1
      FT=Y(K) +(Y(I)-Y(K))*(T-X(K))/(X(I)-X(K))
    4 RETURN
      END
```

```
      SUBROUTINE LAGRA(T,N,TIME,TABLE,ANS1,ANS2,NSWIT)
      DIMENSION TIME(10),TABLE(10)
C
C   THIS ROUTINE FINDS THE INTERPOLATED VALUES OF A FUNCTION TABLE AT
C       THE POINT T AND STORES THE ANSWER IN ANS1.  ALSO THE DERIVATIVE
C       AND STORES THE RESULT IN ANS2.
C        NSWIT IS SET TO 0 FOR BOTH, 1 FOR ONLY FUNCTIONS -1 FOR ONLY DER
C
      NN = N-1
      DO 3 L =2,NN
      IF(T-TIME(L+1))10,3,3
    3 CONTINUE
      L =  N-1
   10 B1 = TIME(L-1) - TIME (L)
      B2 = TIME(L-1) - TIME (L+1)
      B3 = TIME(L)-TIME(L+1)
      A1 = T - TIME(L-1)
      A2 = T - TIME(L)
      A3 = T - TIME(L+1)
      C1 = TABLE(L-1)/(B1*B2)
      C2 =-TABLE(L)/(B1*B3)
      C3 = TABLE(L+1)/(B2*B3)
      IF( NSWIT)  18,15,15
   15 ANS1 = A2*A3*C1 + A1*A3*C2 + A1*A2*C3
      IF( NSWIT) 18,18,19
   18 ANS2 = C1*(A2+A3) + C2*(A1+A3) + C3*(A1 + A2)
   19 RETURN
      END
```

```
      SUBROUTINE AREA
      REAL MU,NEW,MUDOT
      REAL M,LAMDO  ,MM(25),LAMR(25),NT(25),NL(25),LA
      EXTERNAL OUTPU,DERIZ,PASS,DERPR,OUTPR
      DIMENSION PHI(20),TAU(20),F(20,20),FIA(25),
     1 ALT(25),TEMP(25),PRESS(25),RHO(25),HW(25),
     2 VVW(25),ETA(25),TT(25),GR(25)
      DIMENSION C(5)
      COMMON N,H,T,ACCUR,NODUM,REDUC,NODUB,ENDNO,ENDVL,NSAVE,FLAG,Z,VAR
     1(14,8)
      COMMON IUNIT,ISTAN,IALT,ITEMP,IRHO,IM,NPHI,NTAU,NALTS,NWIND,NIM
     1,WS,HG,HO,RE,PHI,TAU,F,FIA,ALT,TEMP,PRESS,RF, HW,VVW,ETA,MU,MUDOT,
     2TT,MM,GR,LAMR,NT,NL,LA,TINIT,GDOT,DTPRN,DELZ1,DELPT,FIMAX,DELFI,PR
     3IN,MTIME,KTIME,M,V,VWX,VWY,GAMR,PSIDT,RHOO,LAMDO,TPRIN,TIME,ZINIT,
     5XM,YM,A,BETA,LUNIT,DADZ,VW,EETA,VWDZ,ETADZ, MUNIT,FI,FA,DEL
      COMMON VWXX,VWYY,UN,UT,P,D,G0,G1,G4,G5
      COMMON SUM,I,DSIM,  C,ZSIMP,IOUT,AGE,IPRIN,ZPRIN
      COMMON G2,G3,G50,G6,G7,G8,G9,G10,G11,G13,H1,H2,H3,H4,H5,TP,H7
      COMMON CF,SF,STH,CTH,THTAO,          CU,SU,CG,SG,NEW,CN,SN,AO,UNO ,
     1UTO,COO,CO,CT,ST,IRH,ARA,ML
      COMMON CON1,CON2,CON3,CON4,L,PRES,DELP,DPMAX
      EQUIVALENCE (PRESS(1),RHO(1)),(Z,TIND)
      NODUM=0
      REDUC=0.
 2000 CALL SSWTCH(6,N)
      GO TO (2001,2002),N
 2001 CALL RESTA
C INTEGRATION INITIALIZATION
 2002 KTIME=KTIME+1
      IF(KTIME-400)  2003,2003,2004
 2004 TIME =TT(NIM)
      GO TO 105
 2003 ML = 1
      N=6
      NSAVE = 1
      ACCUR = 1.
      T = 1.
      NODUB=1
      ENDNO=0.
      ENDVL=0.
      Z=ZINIT
      VAR(1,1)=0.
      VAR(1,2)=0.
      VAR(1,3)=0.
      VAR(1,4) =XM
      VAR(1,5)=YM
      VAR(1,6)=TIME
C
C AREA AND RAY INITIALIZATION
C
      CF=COS(FI*.1745329E-01)
      SF=SIN(FI*.1745329E-01)
      STH=(BETA/M)*CF*COS(GAMR)-(1./M)*SIN(GAMR)
      CTH=SQRT(1.-STH*STH)
      THTAO=ATAN(STH/SQRT(1.-STH*STH))
      CU = COS(MU)
      SU = SIN(MU)
      CG = COS(GAMR)
      SG = SIN(GAMR)
      NEW=LAMDO-ATAN(CU*SF/(SU*CG+CU*CF*SG))
      CN  =COS(NEW)
```

```
      SN   =SIN(NEW)
C     TP IS A TEMPORARY LOACATION
C
C TO FIND THE SPEED OF SOUND AT INITIAL Z
      GO TO (1,2),ISTAN
    1 CALL ATMO(Z,TP,P,D,A0)
      GO TO 3
    2 CALL INPOL(ALT,TEMP,Z,TP)
      A0=49.020576*SQRT(TP)
C
C TO FIND WIND SPEED AT INITIAL Z
C
    3 SINET=SIN(EETA)
      COSET=COS(EETA)
C    TO FIND DU/DZ
      TP   =VWDZ*(SINET*SN+COSET*CN)-VW*ETADZ*(SINET*CN-COSET*SN)
      UNO=VWX *SN+VWY *CN
      UTO=-VWX *CN+VWY *SN
      COO=A0/CTH
      CO=COO-UNO
      G2=1.+SG/(SU*STH)
      G3=CG*CU*SF/CTH
      G50=RHOO*A0*A0
      CTH2=CTH*CTH
      G6=CU/CTH2
      G7=CU*SG+SU*CG*CF
      G8 = STH/(CTH*CTH2)
      G9=CU*STH*SF/CTH2
      G10=STH*SF
      G11=CG*SF/CTH2
      G13=SU*CG+CU*SG*CF
      H1=UTO*PSIDT-(G8*G7*A0-UTO*G11)*MUDOT-(A0*G8*G13-UTO*G9)*GDOT
     1 -(A0*SG/SU)*(TP   -DADZ/CTH)
      TP   =   CU*SF*CG
      H2=PSIDT+G11*MUDOT+G6*G10*GDOT
      H3=-UTO*G6*G7-A0*G8*TP
      H4=-G6*G7
      H5=G3*V+UTO*G2
      H7=H1*H4-H2*H3
      CON1 = (V*TP*G3)/(STH*STH*A0) + (G6*G7*G2* CTH)/STH
      CON1 = 1./( STH*CTH *SQRT(CON1*G50*STH*CTH))
      CON2 = (2.4*CON1)/CO
      SUM=0.
      IOUT=1
      AGE=0.
      IPRIN = 1
      IRH = 2
      ZSIMP=ZINIT
C     FIND ZPRIN THE FIRST PRINT OUT POINT
      DELP = ZINIT/DELPT
      IDEL = DELP
      TP = IDEL
      IF(TP-DELP) 558,559,559
  559 IDEL = IDEL -1
  558 IF(IDEL) 551,552,553
  551 IDEL = 0
  552 IOUT = 2
      ZPRIN = .01
      GO TO 183
  553 ZPRIN= IDEL*DELPT
C     FIND DSIM FOR INITIAL  INTEGRATION STEP
```

```
  183 DELP =(ZINIT - ZPRIN)/2.
  184 DELP =DELP/2.
      IF(DELP-DELZ1)185,185,184
  185 DSIM=-DELP
      H = DSIM
      DELP = (DSIM*.4)/CO
      C(1)=DELP
      C(2)=4.*DELP
      C(3)=2.*DELP
      C(4)=C(2)
      C(5)=C(1)
      I=0
      CALL NODE(DERIZ,PASS,OUTPU,PASS)
      GO TO (310,99,100),IRH
   99  GO TO (300,310),IOUT
  300 IOUT = 2
      GO TO 184
  310 GO TO (30,40) ,ISTAN
   30 CALL ATMO(0.,TP,PRES,D,A)
      GO TO 50
   40 CALL INPOL(ALT,PRESS,0.,PRES)
C     PRESSURE CALCULATIONS
   50 N = 1
      T=3.
      ACCUR=0.
      NODUB = 0
      NSAVE = 1
      TP = ABS(FI*.1745329E-01-FA)
    4 IF(PHI(ML)-TP) 6,5,5
    6 IF (ML-NPHI) 7,5,5
    7 IF(ABS(PHI(ML)-TP)-ABS(PHI(ML+1)-TP)) 5,5,8
    8 ML = ML + 1
      GO TO 4
    5 CON1 = SQRT((G50*CU*CTH2)/SU)
      CON2 = SU/(CO*CTH)
      CON3 = .5*AGE
      CON4 = SQRT(G5/(G4*ARA))
      TIND = TAU(1)*LA
      DPMAX = 0.
      VAR(1,1) = 0.
      VAR(8,1) = 0.
      VAR(1,2) = F(1,ML)
      ENDNO = 0.
      ENDVL = TIND
      L = 1
      CALL OUTPR
      DO 11 L = 2,NTAU
      FLAG = 0.
      ENDVL = TAU(L)*LA
      ENDNO = 4.
      CALL NODE(DERPR,DERPR,OUTPR,PASS)
   11 CONTINUE
      GO TO (110,400,110),IRH
  110 KTIME = KTIME-1
      GO TO 100
  400 WRITE (MUNIT)        FI,VAR(1,4),VAR(1,5),VAR(1,6),DPMAX
  100 FI=FI+DEL
      IF(ABS(FI)-FIMAX) 2000,2000,101
  101 IF(FI) 105,102,102
  102 FI = -DELFI
      DEL = -DEL
      GO TO 2000
  105 RETURN
      END
```

```
      SUBROUTINE DERIZ
      REAL MU,NEW,MUDOT
      REAL M,LAMDO ,MM(25),LAMR(25),NT(25),NL(25),LA
      DIMENSION PHI(20),TAU(20),F(20,20),FIA(25),
     1 ALT(25),TEMP(25),PRESS(25),RHO(25),HW(25),
     2 VVW(25),ETA(25),TT(25),GR(25)
      DIMENSION C(5)
      COMMON N,H,T,ACCUR,NODUM,REDUC,NODUB,ENDNO,ENDVL,NSAVE,FLAG,Z,VAR
     1(14,8)
      COMMON IUNIT,ISTAN,IALT,ITEMP,IRHO,IM,NPHI,NTAU,NALTS,NWIND,NIM
     1,WS,HG,HO,RE,PHI,TAU,F,FIA,ALT,TEMP,PRESS,RF, HW,VVW,ETA,MU,MUDOT,
     2TT,MM,GR,LAMR,NT,NL,LA,TINIT,GDOT,DTPRN,DELZ1,DELPT,FIMAX,DELFI,PR
     3IN,MTIME,KTIME,M,V,VWX,VWY,GAMR,PSIDT,RHOO,LAMDO,TPRIN,TIME,ZINIT,
     5XM,YM,A,BETA,LUNIT,DADZ,VW,EETA,VWDZ,ETADZ, MUNIT,FI,FA,DEL
      COMMON VWXX,VWYY,UN,UT,P,D,G0,G1,G4,G5
      COMMON SUM,I,DSIM,  C,ZSIMP,IOUT,AGE,IPRIN,ZPRIN
      COMMON G2,G3,G50,G6,G7,G8,G9,G10,G11,G13,H1,H2,H3,H4,H5,TP,H7
      COMMON CF,SF,STH,CTH,THTA0,          CU,SU,CG,SG,NEW,CN,SN,A0,UNO ,
     1UTO,COO,CO,CT,ST,IRH,ARA,ML
      EQUIVALENCE (PRESS(1),RHO(1)),(AST,VV),(SC,TP)
      CALL SSWTCH(5,J)
      GO TO (100,4),J
  100 WRITE (LUNIT,155) Z,VAR(1,4),VAR(1,5),VAR(1,6),VAR(1,1),VAR(1,2),V
     1AR(1,3)
  155 FORMAT(3H Z=,E12.4,3H X=,E12.4,3H Y=,E12.4,4H T=  E12.4,4H I1=,E12
     1.4,4H I2= E14.4,4H I3= ,E14.4)
C CALCULATE SPEED OF SOUND,DENSITY,PRESSURE, AND WIND SPEED
    4 IF(Z) 5,6,6
    5 IF(Z-H*.01) 10,10,7
    7 Z = 0.
    6 GO TO(1,2),ISTAN
C     TP  IS A TEMPORARY LOCATION
    1 CALL ATMO(Z,TP,P,D,A)
      GO TO 3
    2 CALL INPOL(ALT,TEMP,Z,TP)
      A=49.020576*SQRT(TP)
      CALL INPOL(ALT,PRESS,Z,P)
      GO TO(11,12),IRHO
   11 D=P/(1716.*TP)
      GO TO 3
   12 D = P
      P=D*(1716.*TP)
    3 CALL LAGRA(Z,NWIND,HW,ETA,TP,UT,1)
      CALL LAGRA(Z,NWIND,HW,VVW,VV,UN,1)
      VWXX=VV*SIN(TP)
      VWYY=VV*COS(TP)
      UN=VWXX*SN+VWYY*CN
      UT=-VWXX*CN+VWYY*SN
      CT=A/(CO+UN)
      ST=SQRT(1.-CT*CT)
      AST=A*ST
      SC=ST/CT
      G0=CT/ST
      G1=G0*G0*G0/(A*A)
C
      VAR(8,1)=-G1
      VAR(8,2)=-G1*UT
      VAR(8,3)=-G1*UT*UT-G0
      VAR(8,4) = -SN/SC+VWXX/AST
      VAR(8,5)=-CN/SC+VWYY/AST
      VAR(8,6)=-1./AST
      VAR(1,7) = CT
      VAR(1,8) = D*A*A
C
   10 RETURN
      END
```

```
      SUBROUTINE OUTPU
      REAL MU,NEW,MUDOT
      REAL M,LAMDO ,MM(25),LAMR(25),NT(25),NL(25),LA
      DIMENSION PHI(20),TAU(20),F(20,20),FIA(25),
     1 ALT(25),TEMP(25),PRESS(25),HW(25),VVW(25),ETA(25),TT(25),GR(25)
      DIMENSION C(5)
      COMMON N,H,T,ACCUR,NODUM,REDUC,NODUB,ENDNO,ENDVL,NSAVE,FLAG,Z,VAR
     1(14,8)
      COMMON IUNIT,ISTAN,IALT,ITEMP,IRHO,IM,NPHI,NTAU,NALTS,NWIND,NIM
     1,WS,HG,HO,RE,PHI,TAU,F,FIA,ALT,TEMP,PRESS,RF, HW,VVW,ETA,MU,MUDOT,
     2TT,MM,GR,LAMR,NT,NL,LA,TINIT,GDOT,DTPRN,DELZ1,DELPT,FIMAX,DELFI,PR
     3IN,MTIME,KTIME,M,V,VWX,VWY,GAMR,PSIDT,RHOO,LAMDO,TPRIN,TIME,ZINIT,
     5XM,YM,A,BETA,LUNIT,DADZ,VW,EETA,VWDZ,ETADZ, MUNIT,FI,FA,DEL
      COMMON VWXX,VWYY,UN,UT,P,D,G0,G1,G4,G5
      COMMON SUM,I,DSIM,  C,ZSIMP,IOUT,AGE,IPRIN,ZPRIN
      COMMON G2,G3,G50,G6,G7,G8,G9,G10,G11,G13,H1,H2,H3,H4,H5,TP,H7
      COMMON CF,SF,STH,CTH,THTAO,          CU,SU,CG,SG,NEW,CN,SN,AO,UNO  ,
     1UTO,COO,CO,CT,ST,IRH,ARA,ML
      COMMON CON1,CON2,CON3,CON4,L,PRES,DELP
      CT = VAR(1,7)
      IF(VAR(1,7)-.9994) 200,210,210
  210 FLAG = -1.
      IRH = 1
      GO TO 6
  200 IF(Z/ZSIMP-1.00001) 1,1,300
    1 I=I+1
C CALCULATE AREA
      ST = SQRT(1.-CT*CT)
      G4=ST*CT
      G5 = VAR(1,8)
      ARA=H3*(VAR(1,2)*G2-VAR(1,1)*H5)-H4*(VAR(1,3)*G2-VAR(1,2)*H5)
     1    +H7*(VAR(1,1)*VAR(1,3)-VAR(1,2)*VAR(1,2))
C CALCULATE INTEGRAND OF AGE FUNCTION AND SUM
      IF (ARA) 410,410,415
  410 TAUA = 0.
      IF(Z-ZINIT)  420,350,350
  350 TP = NEW/.1745329E-01
      WRITE(LUNIT,556) FI,TP ,COO,CO
  556 FORMAT(//30X,*RAY,AREA AND PRESSURE CALCULATIONS*//
     110X,3HFI=,F10.3,10X,3HNU=,F10.3,10X,4HCOO=,F10.3,7X,3HCO=,F10.3//
     26X,1HX,15X,1HY,15X,1HZ,15X,1HT,12X,4HAREA,11X,3HAGE,10X,9HCOS THET
     1A)
      GO TO 50
  430 IRH = 4
      GO TO 421
  420 IRH = 3
  421 FLAG = -1.
      AGE = -SUM+CON2*SQRT(ZINIT-Z)
      GO TO 6
  415 TP = SQRT(ZINIT-Z)
      IF(G4*G5)  430,430,427
  427 TAUA=1./SQRT(G5*ARA*G4*G4*G4)-CON1/TP
  416 SUM= SUM+C(I)*TAUA
  401 IF(I-5)50,5,50
    5 AGE = -SUM+CON2*TP
      SUM=C(1)*TAUA+ SUM
      I=1
      HXS=VAR(1,4)
      HYS=VAR(1,5)

      HZS=Z
      HTS=VAR(1,6)
```

```
      AREAS=ARA
      CTS=CT
C CHECK TO SEE IF AT ZERO OR PRIN OR PRINT INTERVAL
      IF(Z/ZPRIN-1.00001) 66,66,9
   21 IPRIN = 2
    6 VAR(1,4)=HXS
      VAR(1,5)=HYS
      Z=HZS
      VAR(1,6)=HTS
      ARA=AREAS
      CT=CTS
C     SAVE VARIABLES FOR USE IF THETA OR AREA APPROACH ZERO
C     RESTORE VARIABLES WHEN THETA OR AREA APPROACH ZERO
C
C
      WRITE(LUNIT,61)VAR(1,4),VAR(1,5),Z,VAR(1,6),ARA,AGE,CT
   61 FORMAT(1X,7(E13.5,2X))
   68 IF(Z-.1) 10,10,100
   10 FLAG = -1.
      GO TO (88,300,77,99),IRH
   99 WRITE(LUNIT,90)
   90 FORMAT(/20X,27HNEGATIVE SQUARE ROOT IN AGE )
      IRH = 3
      RETURN
   77 WRITE(LUNIT,70)
   70 FORMAT(/20X,9HAREA ZERO)
      RETURN
   88 WRITE(LUNIT,80)
   80 FORMAT(/20X,14HRAY HORIZONTAL)
      RETURN
    9 GO TO (8,50),IPRIN
    8 IF(Z-PRIN) 21,21,50
   66 ZPRIN = ZPRIN -DELPT
      IF(ZPRIN) 64,64,65
   64 ZPRIN = .1
   65 IF(Z- PRIN) 67,67,6
   67 IPRIN = 2
      GO TO 6
  100 GO TO (14,50),IOUT
   14 DELP=DELPT/2.
      GO TO 10
   50 ZSIMP=ZSIMP+DSIM
      GO TO  (88,52,77),IRH
   52 IF(ZSIMP-1.) 51,51,300
C   ADJUST ZSIMP SO THAT DIVIDE BY ZERO DOES NOT OCCUR
   51 ZSIMP = .1
  300 RETURN
      END
```

```
      SUBROUTINE DERPR
      REAL M,LAMDO ,MM(25),LAMR(25),NT(25),NL(25),LA,MU,MUDOT,NEW
      DIMENSION PHI(20),TAU(20),F(20,20),FIA(25),C(5),
     1 ALT(25),TEMP(25),PRESS(25),HW(25),VVW(25),ETA(25),TT(25),GR(25)
      COMMON NNODE,HNODE,TNODE,ACC,NODUM,BYHAL ,NODUB,ENDNO,ENDVL,NSAVE,
     5FLAG,TIND,VAR(14,8)
      COMMON IUNIT,ISTAN,IALT,ITEMP,IRHO,IM,NPHI,NTAU,NALTS,NWIND,NIM
     1,WS,HG,HO,RE,PHI,TAU,F,FIA,ALT,TEMP,PRESS,RF, HW,VVW,ETA,MU,MUDOT,
     2TT,MM,GR,LAMR,NT,NL,LA,TINIT,GDOT,DTPRN,DELZ1,DELPT,FIMAX,DELFI,PR
     3IN,MTIME,KTIME,M,V,VWX,VWY,GAMR,PSIDT,RHOO,LAMDO,TPRIN,TIME,ZINIT,
     5XM,YM,A,BETA,LUNIT,DADZ,VW,EETA,VWDZ,ETADZ, MUNIT,FI,FA,DEL
      COMMON VWXX,VWYY,UN,UT,P,D,G0,G1,G4,G5
      COMMON SUM,I,DSIM,  C,ZSIMP,IOUT,AGE,IPRIN,ZPRIN
      COMMON G2,G3,G50,G6,G7,G8,G9,G10,G11,G13,H1,H2,H3,H4,H5,TP,H7
      COMMON CF,SF,STH,CTH,THTAO,CU,SU,CG,SG,NEW,CN,SN,AO,UNO,UTO,COO,CO
     1,CT,ST,IRH,ARA,ML
      COMMON CON1,CON2,CON3,CON4,L
      TP = TIND/LA
      CALL INPOL(TAU,F(1,ML),TP,VAR(1,2))
      VAR(8,1) = CON1*VAR(1,2)
      CALL SSWTCH(5,J)
      GO TO (100,1),J
  100 WRITE(LUNIT,155) TIND,VAR(1,1),VAR(8,1)
  155 FORMAT( 3H Z=  E15.4,9HINTEGRAL=  E14.5,10HINTEGRAND= E15.4)
    1 RETURN
      END

      SUBROUTINE OUTPR
      REAL M,LAMDO ,MM(25),LAMR(25),NT(25),NL(25),LA,MU,MUDOT,NEW
      DIMENSION PHI(20),TAU(20),F(20,20),FIA(25),C(5),
     1 ALT(25),TEMP(25),PRESS(25),HW(25),VVW(25),ETA(25),TT(25),GR(25)
      COMMON NNODE,HNODE,TNODE,ACC,NODUM,BYHAL ,NODUB,ENDNO,ENDVL,NSAVE,
     5FLAG,TIND,VAR(14,8)
      COMMON IUNIT,ISTAN,IALT,ITEMP,IRHO,IM,NPHI,NTAU,NALTS,NWIND,NIM
     1,WS,HG,HO,RE,PHI,TAU,F,FIA,ALT,TEMP,PRESS,RF, HW,VVW,ETA,MU,MUDOT,
     2TT,MM,GR,LAMR,NT,NL,LA,TINIT,GDOT,DTPRN,DELZ1,DELPT,FIMAX,DELFI,PR
     3IN,MTIME,KTIME,M,V,VWX,VWY,GAMR,PSIDT,RHOO,LAMDO,TPRIN,TIME,ZINIT,
     5XM,YM,A,BETA,LUNIT,DADZ,VW,EETA,VWDZ,ETADZ, MUNIT,FI,FA,DEL
      COMMON VWXX,VWYY,UN,UT,P,D,G0,G1,G4,G5
      COMMON SUM,I,DSIM,  C,ZSIMP,IOUT,AGE,IPRIN,ZPRIN
      COMMON G2,G3,G50,G6,G7,G8,G9,G10,G11,G13,H1,H2,H3,H4,H5,TP,H7
      COMMON CF,SF,STH,CTH,THTAO,CU,SU,CG,SG,NEW,CN,SN,AO,UNO,UTO,COO,CO
     1,CT,ST,IRH,ARA,ML
      COMMON CON1,CON2,CON3,CON4,L,PRES,DELP,DPMAX
      IF(L-1) 5,5,6
    5 WRITE (LUNIT,200)
  200 FORMAT(//5X,1HL,10X,1HF,11X,5HVE(L),7X,4HXI 0,8X,4HXI 1,8X,5HS INT
     1,7X,1HS,11X,2HP1,10X,5HDEL P)
    6 IF(ENDNO-4.) 10,20,20
   10 XIO = TIND*CON2
      XI1 = XIO-AGE*VAR(8,1)
      SINT = CON2*VAR(1,1)
      S = SINT     -CON3*VAR(8,1)*VAR(8,1)
      DELP = VAR(8,1)*CON4*RF
      IF(ABS(DELP)-DPMAX) 50,50,60
   60 DPMAX =  ABS(DELP)
   50 P1 = DELP/PRES
      WRITE (LUNIT,2)TIND,VAR(1,2),VAR(8,1),XIO,XI1, SINT,S,P1,DELP
    2 FORMAT (9F12.6)
      IF(ENDNO) 30,30,20
   30 FLAG = -1.
   20 RETURN
      END
```

```
      SUBROUTINE NODE(COMPD ,COMPY,COMPT,COMPE )
      DIMENSION VAR(14,1) ,A(4),B(4),C(4)
      COMMON N,H,T,PHSIG,NODUM,BYHAL ,NODUB,ENDNO,ENDVA ,NSAVE,FLAG,X,
     1VAR
C     INITIALIZE
      MODF(XSAVE,PERR) = XSAVE -FLOAT(IFIX(XSAVE/PERR))*PERR
      FLAG=0.
  500 IF (BYHAL )502,501,502
  501 BYHAL =.5
  502 IF (ENDNO)503,504,503
C     ENDPOINT COMPUTES H
  503 H=(ENDVA -X)/ENDNO
C     PREPARE FOR RKG
  504 CALL COMPD
  505 CALL COMPT
      IF (FLAG)560,506,506
  506 XSAVE=X
C     PROGRAM USING RKG AS A STARTING PROCEDURE
      IF(A(1)-.5) 400,401,400
  400 A(1)=.5
      A(2)=.29289322
      A(3)=1.7071068
      A(4)=.16666667
      B(1)=1.
      B(2)=.29289322
      B(3)=1.7071068
      B(4)=.33333333
      C(1)=.5
      C(2)=.29289322
      C(3)=1.7071068
      C(4)=.5
  401 DO 402 I=1,N
  402 VAR(6,I)=0.
      J=4
      GO TO 410
  403 DO 407 K=1,4
      DO 404 I=1,N
      CK=H*VAR(8,I)
      R=(A(K)*CK)-(B(K)*VAR(6,I))
      VAR(1,I)=VAR(1,I)+R
  404 VAR(6,I)=VAR(6,I)+(3.*R)-(C(K)*CK)
      IF (K-1)405,405,413
  413 IF(K-3) 406,405,406
C     NEW VALUE OF X
  405 X=X+(H/2.)
      CALL COMPD
      GO TO 407
  406 CALL COMPY
  407 CONTINUE
      IF (NODUM)410,412,411
  412 NODUM=-1
  410 NN=N+NSAVE
      DO 408 I=1,NN
      VAR(J+1,I)=VAR(1,I)
  408 VAR(J+8,I)=VAR(8,I)
      J=J-1
      IF (J) 409,409,403
  411 CALL COMPT
      IF(FLAG)560,410,410
  409 NSWHF=1
      IF (ENDNO)507,508,507
```

```
  507 ENDNO=ENDNO-3.
  508 M=3
  509 FLAG=0.
  510 X=X+H
C     PROGRAM FOR THE PREDICTOR
      DO 450 I=1,N
  450 VAR(1,I)=(1.5476511*VAR(2,I))-(1.8675052*VAR(3,I))+(2.0172069*
     1VAR(4,I))-(.6973528*VAR(5,I))+H*((2.0022473*VAR(9,I))-(2.0316877*
     2VAR(10,I))+(1.8186108*VAR(11,I))-(.71432005*VAR(12,I)))
  512 CALL COMPD
  513 PERR=0.
C     PROGRAM FOR THE CORRECTOR
      DO 462 I=1,N
  460 TEMP=VAR(2,I)+H*((.375*VAR(8,I))+(.79166667*VAR(9,I))
     1-(.20833333*VAR(10,I))+(.041666667*VAR(11,I)))
      IF (PHSIG-1.)    463,464,463
  463 TEMPA=ABS ((TEMP-VAR(1,I))/TEMP)
      GO TO 465
  464 TEMPA=ABS (TEMP-VAR (1,I))
  465 VAR(1,I)=TEMP
      IF (PERR-TEMPA)461,462,462
  461 PERR=TEMPA
  462 CONTINUE
  515 CALL COMPY
  516 IF (PERR-16.219659*10.**(-T))517,517,535
C     NO HALVING NECESSARY
  517 NSWHF=0
      IF (NODUM)550,518,518
  518 IF (ENDNO)519,520,519
  519 ENDNO=ENDNO-1.
  520 CALL COMPT
      IF (FLAG)560,521,521
C     IS DOUBLING POSSIBLE
  521 IF (PERR-(( 16.219659* 10.**(-T))/200.)) 525,525,522
  522 M=3
  528 J=13
  523 DO 524 I=1,N
  524 VAR(J+1,I)=VAR(J,I)
      J=J-1
      IF (J) 509,509,523
C     DOUBLING
  525 M=M-1
  526 IF (M) 530,527,528
  527 IF (ENDNO)530,531,530
  530 IF (MODF(ENDNO,2.))528,531,528
  531 FLAG=2.
      CALL COMPE
      IF(FLAG)560,532,532
  532 IF(NODUB)522,529,522
  529 NN=N+NSAVE
      DO 533 I=1,NN
      VAR(2,I)=VAR(1,I)
      VAR(4,I)=VAR(5,I)
      VAR(5,I)=VAR(7,I)
      VAR(9,I)=VAR(8,I)
      VAR(11,I)=VAR(12,I)
  533 VAR(12,I)=VAR(14,I)
      H=2.*H
      IF (ENDNO)534,508,534
  534 ENDNO=ENDNO/2.
      GO TO 508
```

```
C       HALVING
  535 FLAG=ABS (BYHAL )
      CALL COMPE
  565 IF (FLAG)560,561,561
  561 IF (NODUM)537,537,536
  536 CALL COMPT
      IF (FLAG)560,537,537
  537 IF (BYHAL -1.)548,517,517
  548 IF(ENDNO-1.) 542,563,543
  563 ENDNO= 2.*ENDNO
      H=H/2.
      FLAG=.5
      CALL COMPE
      IF (FLAG) 560,543,543
  543 ENDNO=ENDNO/BYHAL
      NN=N+NSAVE
  542 IF (NSWHF)538,540,538
C       REPEATED HALVING
  538 DO 539 I=1,NN
      VAR(1,I)=VAR(5,I)
  539 VAR(8,I)=VAR(12,I)
      X=XSAVE
      IF(ENDNO)549,549,544
  544 ENDNO=ENDNO+3./BYHAL
  549 H=H*ABS (BYHAL )
      GO TO 506
  540 DO 541 I=1,NN
      VAR(1,I)=VAR(2,I)
  541 VAR(8,I)=VAR(9,I)
      X=X-H
      GO TO 549
C       DUMMY OUTPUTTING
  550 X=XSAVE+H
      IF (ENDNO)551,552,551
  551 ENDNO=ENDNO+2.
  552 K=3
      NN=N+NSAVE
      DO 553 I=1,NN
      VAR(6,I)=VAR(1,I)
  553 VAR(13,I)=VAR(8,I)
  557 DO 554 I=1,NN
      VAR(1,I)=VAR(K+1,I)
  554 VAR(8,I)=VAR(K+8,I)
      CALL COMPT
      IF(FLAG)560,562,562
  560 RETURN
  562 X=X+H
      K=K-1
      IF (K) 558,558,555
  555 IF(ENDNO)556,557,556
  556 ENDNO=ENDNO-1.
      GO TO 557
  558 DO 559 I=1,NN
      VAR(1,I)=VAR(6,I)
  559 VAR(8,I)=VAR(13,I)
      NODUM=0
      GO TO 518
      END
```

```
      SUBROUTINE PASS
      RETURN
      END


      SUBROUTINE GROUN
C     ROUTINE FOR INTERPOLATION OF GROUND  INTERSECTIONS
C
      COMMON NRMAX,HN,TN,AC,MUNIT,BY,LUNIT,T(400),X(400),Y(400),P(400),
     1N(41)
      I = 1
      K = 1
      NPHSW = 1
      WRITE (LUNIT,667)
  667 FORMAT(1H1,20X,34HLIST OF SAVED GROUND INTERSECTIONS)
      DO 150 NREC = 1,NRMAX
C  READING A RECORD FROM THE DISK OR TAPE
   30 READ(MUNIT)            FI,X(I),Y(I),T(I),P(I)
      WRITE (LUNIT,666) FI,X(I),Y(I),T(I),P(I),I
  666 FORMAT (5F20.5,2I10)
      IF(FI) 50,200,100
   50 GO TO (51,100),NMNSW
C    FIRST NEGATIVE ANGLE
   51 NMNSW = 2
      CALL MOVE(I,I+1)
      CALL MOVE(IMIN,I)
      N(K) = I
      K = K + 1
      I = I +1
C    FIND THE TIME MINIMUM FOR CURVE
  100 IF(T(I) - TMIN) 110,150,150
  110 TMIN = T(I)
      J = I
      GO TO 150
C    FI IS EQUAL TO ZERO
  200 GO TO (201,210),NPHSW
  201 NPHSW = 2
  220 N(K) = I
      K = K + 1
      IMIN = I
      NMNSW = 1
      GO TO 110
C   CHECK IF TMIN IS AT FI = 0
  210 GO TO (211,215),NMNSW
  211 KI = I+1
      CALL MOVE(I,KI)
      CALL MOVE(IMIN,I)
      N(K) = I
      K = K + 1
      I = I+1
  215 IF(J-IMIN) 1000,220,230
  230 ISAVE = I+1
      JMAX = N(K-1)-1
      LOOPS = IMIN +1
C
C     MOVE TABLES OUT OF PERMUTATION AREA
C
```

```
      DO 231 KI = LOOPS,JMAX
      CALL MOVE(KI,ISAVE)
  231 ISAVE = ISAVE + 1
C      MOVING THE TABLES AROUND
      IF(J-N(K-1))240,1000,250
  240 KK = J-IMIN-1
      LOOPS = I + KK +1
      IC = 1
      N(K-1) = JMAX - KK
      DO 244 KI = IMIN,JMAX
      CALL MOVE(LOOPS,KI)
      LOOPS = LOOPS + IC
      IF(LOOPS-ISAVE) 244,242,242
  242 LOOPS = I + KK +1
      IC = -1
  244 CONTINUE
      GO TO 220
  250 JMAX = J-1
      IC = -1
      LOOPS = J
      DO 254 KI=IMIN,JMAX
      CALL MOVE(LOOPS,KI)
      LOOPS = LOOPS + IC
      IF(LOOPS - N(K-1))  252,254,254
  252 LOOPS = I +1
      IC = 1
  254 CONTINUE
      N(K-1) = J
      GO TO 220
  150 I = I + 1
      KMAX = K-2
      WRITE(LUNIT,413)
      DO 450 K = 3,KMAX,2
      WRITE (LUNIT,412)
      KN = N(K)
      TVAL = T(KN)
      KI = K + 1
      KK = 1
      IC = 2
  400 CALL INTEP(TVAL,XVAL,YVAL,PVAL,KK,KERR)
      GO TO (405,499),KERR
  405 WRITE(LUNIT,410) TVAL,XVAL,YVAL,PVAL
  410 FORMAT(4F20.5)
  406 KK = KK +IC
      IF(KK-KI) 420,450,411
  420  IF(KK) 450,450,400
  411 KK = K - 1
      IC = -2
      GO TO 400
  222 FORMAT(* NO DATA POINT ON CURVE * I5)
  499 WRITE(LUNIT,222) KK
      GO TO 406
  450 CONTINUE
   20 WRITE (LUNIT,60)
   60 FORMAT(//20X,17HPROBLEM COMPLETED)
      RETURN
  412 FORMAT(/10X,4HTIME,16X,1HX,19X,1HY,19X,8HPRESSURE)
  413 FORMAT(1H1,20X,26HGROUND INTERSECTION CURVES)
 1000 WRITE(LUNIT,1001)
 1001 FORMAT(13H KULSRUD GOOF)
      CALL EXIT
      END
```

```
      SUBROUTINE INTEP(TVAL,XANS,YANS,PANS,KK,NERR)
C     AN INTERPOLATION ROUTINE IN THE SPECIAL TABLES
      COMMON NRMAX,HN,TN,AC,MUNIT,BY,LUNIT,T(400),X(400),Y(400),P(400),
     1N(41)
      IN = N(KK)
      INF = N(KK+1)-1
      DO 200  IS = IN,INF
      IF(T(IS)-TVAL) 200,100,300
  100 PERCT = 1.
      GO TO 302
  200 CONTINUE
  201 NERR = 2
      RETURN
  300 IF(IS-IN) 201,201,301
  301 PERCT = (TVAL-T(IS-1))/(T(IS)-T(IS-1))
  302 XANS = PERCT*(X(IS)-X(IS-1))+ X(IS-1)
      YANS = PERCT*(Y(IS)-Y(IS-1))+ Y(IS-1)
      PANS = PERCT*(P(IS)-P(IS-1))+ P(IS-1)
      NERR = 1
      RETURN
      END
```

```
      SUBROUTINE MOVE(K,J)
      COMMON NRMAX,HN,TN,AC,MUNIT,BY,LUNIT,T(400),X(400),Y(400),P(400),
     1N(41)
      T(J) = T(K)
      X(J) = X(K)
      Y(J) = Y(K)
      P(J) = P(K)
      RETURN
      END
```

```
      SUBROUTINE RESTA
      COMMON C(1000)
      REWIND 9
      WRITE (9) C
      REWIND 9
      WRITE (6,1)
    1 FORMAT(/23HOTERMINATED BY OPERATOR
      CALL EXIT
    2 RETURN
      END
```

```
      SUBROUTINE INITAL
      COMMON C(1000)
      REWIND 9
      READ (9) C
      REWIND 9
      RETURN
      END
```

# APPENDIX B SAMPLE PRINTOUT

AERO RESEARCH ASSOC OF PRINCETON    TEST PROBLEM  M=1.5  H=30000. FT

IUNIT= 1    ISTAN= 1    IALT= 1    ITEMP= 1    IRHO= 1    IM= 1

NPHI= 2    NTAU= 19    NALTS= 6    NWIND= 3    NIM= 4

WING LOADING= 50.000    AIRPLANE LENGTH= 100.000

HG= 1500.    HO= 30000.    RE= 0.2089007E 08

| PHI | L/LA | F |
|---|---|---|
| 0.00 | 0.00 | 0.00000 |
| 0.00 | 0.04 | 0.01110 |
| 0.00 | 0.10 | 0.01600 |
| 0.00 | 0.13 | 0.01500 |
| 0.00 | 0.20 | 0.01220 |
| 0.00 | 0.30 | 0.00450 |
| 0.00 | 0.34 | 0.00300 |
| 0.00 | 0.40 | 0.01100 |
| 0.00 | 0.44 | 0.00000 |
| 0.00 | 0.50 | -0.03400 |
| 0.00 | 0.56 | -0.02000 |
| 0.00 | 0.60 | -0.00900 |
| 0.00 | 0.65 | -0.01300 |
| 0.00 | 0.70 | -0.01500 |
| 0.00 | 0.75 | -0.01300 |
| 0.00 | 0.82 | -0.01000 |
| 0.00 | 0.92 | 0.00000 |
| 0.00 | 1.00 | 0.01000 |
| 0.00 | 1.12 | 0.00020 |
| 45.00 | 0.00 | 0.00000 |
| 45.00 | 0.04 | 0.01110 |
| 45.00 | 0.10 | 0.01600 |
| 45.00 | 0.13 | 0.01500 |
| 45.00 | 0.20 | 0.01220 |
| 45.00 | 0.30 | 0.00450 |
| 45.00 | 0.34 | 0.00300 |
| 45.00 | 0.40 | 0.01100 |
| 45.00 | 0.44 | 0.00000 |
| 45.00 | 0.50 | -0.03400 |
| 45.00 | 0.56 | -0.02000 |
| 45.00 | 0.60 | -0.00555 |
| 45.00 | 0.65 | -0.01300 |
| 45.00 | 0.70 | -0.01500 |
| 45.00 | 0.75 | -0.01300 |
| 45.00 | 0.82 | -0.01000 |
| 45.00 | 0.92 | 0.00000 |
| 45.00 | 1.00 | 0.01000 |
| 45.00 | 1.12 | 0.00020 |

A

| ALTITUDE | WIND SPEED | DIRECTION |
|---|---|---|
| 0. | 0.00 | 270.0 |
| 20000. | 0.00 | 270.0 |
| 100000. | 0.00 | 270.0 |

| TIME | NT | NL | MACH NO | GAMMA | PSI | BANK ANGLE |
|---|---|---|---|---|---|---|
| 1000.000 | 0.200 | 1.800 | 1.500 | 1.000 | 90.000 | 0.000 |
| 1001.000 | 0.200 | 1.800 | 1.508 | 1.010 | 90.500 | 0.000 |
| 1002.000 | 0.200 | 1.800 | 1.516 | 1.020 | 91.000 | 0.000 |
| 1003.000 | 0.200 | 1.800 | 1.525 | 1.030 | 91.500 | 0.000 |

A

INITIAL TIME= 1000.000 DELTA T PRINT= 1.00000

DELTA Z= 300.00006 PRINT INTERVAL= 2000.000

PRINT OUT POINT= 3500.000 DPHI= 20.000 MAXIMUM PHI= 60.000

REFLECTION FACTOR = 1.800 F FACTOR = 3.784

MANEUVER DATA - OPTION ONE

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| TIME | = | 1000.0 | H | = | 30000.0 | | | | | | | | | | | | |
| XG | = | 0.0 | YG | = | 0.0 | ZG | = | 28500.0 | GAMMA | = | 0.99 | PSI | = | 89.99 | MU | = | 41.81 |
| XGDOT | = | 1491.4 | YGDOT | = | 0.0 | ZGDOT | = | 26.0 | GAMDOT | = | 0.98 | PSIDOT | = | 0.00 | MUDOT | = | -0.2206 |
| CT-CD | = | 0.010090 | CL | = | 0.090815 | M | = | 1.50 | Q | = | 991.0 | V | = | 1491.7 | VDOT | = | 5.87 |
| UX | = | 0.0000 | UY | = | 0.0000 | A | = | 994.47 | | | | VG | = | 1491.7 | | | |

B

RAY,AREA AND PRESSURE CALCULATIONS

FI= 0.000 NU= 90.000 COO= 1463.380 CO= 1463.380

| X | Y | Z | T | AREA | AGE | COS THETA |
|---|---|---|---|---|---|---|
| 0.46410E 03 | 0.29601E-03 | 0.28000E 05 | 0.10006E 04 | 0.53164E 03 | 0.33891E-02 | 0.68103E 00 |
| 0.23392E 04 | 0.14920E-02 | 0.26000E 05 | 0.10034E 04 | 0.27713E 04 | 0.74020E-02 | 0.68686E 00 |
| 0.42445E 04 | 0.27073E-02 | 0.24000E 05 | 0.10061E 04 | 0.52002E 04 | 0.97310E-02 | 0.69265E 00 |
| 0.61806E 04 | 0.39422E-02 | 0.22000E 05 | 0.10088E 04 | 0.78299E 04 | 0.11459E-01 | 0.69838E 00 |
| 0.81478E 04 | 0.51969E-02 | 0.20000E 05 | 0.10116E 04 | 0.10672E 05 | 0.12843E-01 | 0.70407E 00 |
| 0.10146E 05 | 0.64719E-02 | 0.18000E 05 | 0.10143E 04 | 0.13742E 05 | 0.13997E-01 | 0.70972E 00 |
| 0.12177E 05 | 0.77674E-02 | 0.16000E 05 | 0.10170E 04 | 0.17051E 05 | 0.14982E-01 | 0.71532E 00 |
| 0.14241E 05 | 0.90837E-02 | 0.14000E 05 | 0.10198E 04 | 0.20616E 05 | 0.15836E-01 | 0.72088E 00 |
| 0.16338E 05 | 0.10421E-01 | 0.12000E 05 | 0.10225E 04 | 0.24451E 05 | 0.16587E-01 | 0.72640E 00 |
| 0.18469E 05 | 0.11780E-01 | 0.10000E 05 | 0.10253E 04 | 0.28575E 05 | 0.17253E-01 | 0.73188E 00 |
| 0.20634E 05 | 0.13161E-01 | 0.80000E 04 | 0.10280E 04 | 0.33004E 05 | 0.17848E-01 | 0.73732E 00 |
| 0.22835E 05 | 0.14565E-01 | 0.60000E 04 | 0.10307E 04 | 0.37760E 05 | 0.18384E-01 | 0.74271E 00 |
| 0.25071E 05 | 0.15991E-01 | 0.40000E 04 | 0.10335E 04 | 0.42861E 05 | 0.18867E-01 | 0.74808E 00 |
| 0.26203E 05 | 0.16713E-01 | 0.30000E 04 | 0.10349E 04 | 0.45548E 05 | 0.19092E-01 | 0.75074E 00 |
| 0.27344E 05 | 0.17441E-01 | 0.20000E 04 | 0.10362E 04 | 0.48331E 05 | 0.19307E-01 | 0.75340E 00 |
| 0.29655E 05 | 0.18915E-01 | 0.00000E 00 | 0.10390E 04 | 0.54193E 05 | 0.19707E-01 | 0.75869E 00 |

C

| L | F | VE(L) | XI 0 | XI 1 | S INT | S | P1 | DEL P |
|---|---|---|---|---|---|---|---|---|
| 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 1.125000 | 0.010500 | 0.223945 | 0.000754 | -0.003659 | 0.000084 | -0.000409 | 0.000065 | 0.130412 |
| 2.250000 | 0.021001 | 0.447890 | 0.001508 | -0.007318 | 0.000337 | -0.001638 | 0.000130 | 0.260825 |
| 3.374999 | 0.031501 | 0.671835 | 0.002262 | -0.010978 | 0.000760 | -0.003687 | 0.000195 | 0.391237 |
| 4.499999 | 0.042002 | 0.895780 | 0.003016 | -0.014637 | 0.001351 | -0.006555 | 0.000260 | 0.521650 |
| 5.874999 | 0.046637 | 0.994639 | 0.003938 | -0.015663 | 0.002222 | -0.007526 | 0.000289 | 0.579220 |
| 7.249999 | 0.051273 | 1.093498 | 0.004860 | -0.016690 | 0.003184 | -0.008598 | 0.000317 | 0.636789 |
| 8.625000 | 0.055908 | 1.192357 | 0.005781 | -0.017717 | 0.004238 | -0.009771 | 0.000346 | 0.694359 |
| 10.000000 | 0.060543 | 1.291216 | 0.006703 | -0.018743 | 0.005382 | -0.011046 | 0.000375 | 0.751929 |
| 10.875000 | 0.059597 | 1.271040 | 0.007290 | -0.017759 | 0.006134 | -0.009785 | 0.000369 | 0.740180 |
| 11.750000 | 0.058651 | 1.250865 | 0.007876 | -0.016775 | 0.006874 | -0.008544 | 0.000363 | 0.728431 |
| 12.625000 | 0.057705 | 1.230690 | 0.008463 | -0.015791 | 0.007601 | -0.007323 | 0.000357 | 0.716682 |
| 13.500000 | 0.056759 | 1.210515 | 0.009050 | -0.014806 | 0.008317 | -0.006121 | 0.000351 | 0.704933 |
| 15.124998 | 0.054111 | 1.154024 | 0.010139 | -0.012604 | 0.009605 | -0.003517 | 0.000335 | 0.672036 |
| 16.749996 | 0.051462 | 1.097534 | 0.011228 | -0.010401 | 0.010832 | -0.001037 | 0.000318 | 0.639140 |
| 18.374992 | 0.048813 | 1.041043 | 0.012318 | -0.008198 | 0.011996 | 0.001317 | 0.000302 | 0.606243 |
| 19.999988 | 0.046164 | 0.984553 | 0.013407 | -0.005996 | 0.013100 | 0.003548 | 0.000286 | 0.573346 |
| 22.499988 | 0.038880 | 0.829203 | 0.015083 | -0.001258 | 0.014620 | 0.007844 | 0.000240 | 0.482880 |
| 24.999988 | 0.031596 | 0.673854 | 0.016759 | 0.003478 | 0.015879 | 0.011405 | 0.000195 | 0.392413 |
| 27.499988 | 0.024312 | 0.518504 | 0.018435 | 0.008216 | 0.016878 | 0.014229 | 0.000150 | 0.301947 |
| 29.999988 | 0.017028 | 0.363155 | 0.020111 | 0.012954 | 0.017617 | 0.016317 | 0.000105 | 0.211480 |
| 30.999988 | 0.015609 | 0.332892 | 0.020781 | 0.014220 | 0.017850 | 0.016758 | 0.000096 | 0.193856 |
| 31.999988 | 0.014190 | 0.302629 | 0.021451 | 0.015487 | 0.018063 | 0.017161 | 0.000087 | 0.176233 |
| 32.999992 | 0.012771 | 0.272366 | 0.022122 | 0.016754 | 0.018256 | 0.017525 | 0.000079 | 0.158610 |

D

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 33.999992 | 0.011352 | 0.242103 | 0.022792 | 0.018021 | 0.018428 | 0.017851 | 0.000070 | 0.140987 |
| 35.499992 | 0.018919 | 0.403503 | 0.023798 | 0.015845 | 0.018753 | 0.017149 | 0.000117 | 0.234977 |
| 36.999992 | 0.026487 | 0.564905 | 0.024803 | 0.013670 | 0.019240 | 0.016095 | 0.000164 | 0.328968 |
| 38.499992 | 0.034055 | 0.726307 | 0.025809 | 0.011495 | 0.019889 | 0.014691 | 0.000211 | 0.422959 |
| 39.999992 | 0.041623 | 0.887709 | 0.026814 | 0.009319 | 0.020701 | 0.012935 | 0.000257 | 0.516950 |
| 40.999992 | 0.031218 | 0.665786 | 0.027485 | 0.014363 | 0.021221 | 0.016853 | 0.000193 | 0.387715 |
| 41.999992 | 0.020812 | 0.443858 | 0.028155 | 0.019407 | 0.021593 | 0.019652 | 0.000128 | 0.258477 |
| 42.999992 | 0.010406 | 0.221931 | 0.028825 | 0.024452 | 0.021816 | 0.021331 | 0.000064 | 0.129240 |
| 43.999992 | 0.000000 | 0.000004 | 0.029496 | 0.029496 | 0.021891 | 0.021891 | 0.000000 | 0.000002 |
| 45.499992 | -0.032163 | -0.685949 | 0.030501 | 0.044020 | 0.021546 | 0.016909 | -0.000199 | -0.399457 |
| 46.999992 | -0.064327 | -1.371907 | 0.031507 | 0.058544 | 0.020511 | 0.001965 | -0.000398 | -0.798918 |
| 48.499992 | -0.096491 | -2.057868 | 0.032512 | 0.073069 | 0.018787 | -0.022942 | -0.000597 | -1.198382 |
| 49.999992 | -0.128655 | -2.743825 | 0.033518 | 0.087593 | 0.016373 | -0.057813 | -0.000797 | -1.597844 |
| 51.499992 | -0.115412 | -2.461385 | 0.034524 | 0.083033 | 0.013756 | -0.045943 | -0.000715 | -1.433367 |
| 52.999992 | -0.102168 | -2.178931 | 0.035529 | 0.078472 | 0.011423 | -0.035361 | -0.000633 | -1.268882 |
| 54.499992 | -0.088924 | -1.896475 | 0.036535 | 0.073910 | 0.009374 | -0.026067 | -0.000551 | -1.104397 |
| 55.999992 | -0.075680 | -1.614022 | 0.037540 | 0.069349 | 0.007608 | -0.018061 | -0.000469 | -0.939912 |
| 56.999992 | -0.065274 | -1.392094 | 0.038211 | 0.065646 | 0.006601 | -0.012494 | -0.000404 | -0.810675 |
| 57.999992 | -0.054868 | -1.170167 | 0.038881 | 0.061943 | 0.005742 | -0.007750 | -0.000340 | -0.681437 |
| 58.999992 | -0.044462 | -0.948239 | 0.039551 | 0.058239 | 0.005032 | -0.003827 | -0.000275 | -0.552199 |
| 59.999992 | -0.034056 | -0.726311 | 0.040222 | 0.054536 | 0.004471 | -0.000727 | -0.000211 | -0.422961 |
| 61.249992 | -0.037839 | -0.807009 | 0.041060 | 0.056964 | 0.003828 | -0.002588 | -0.000234 | -0.469955 |
| 62.499992 | -0.041623 | -0.887709 | 0.041898 | 0.059393 | 0.003118 | -0.004646 | -0.000257 | -0.516950 |
| 63.749992 | -0.045407 | -0.968411 | 0.042736 | 0.061821 | 0.002341 | -0.006900 | -0.000281 | -0.563946 |
| 65.000000 | -0.049191 | -1.049111 | 0.043574 | 0.064249 | 0.001495 | -0.009349 | -0.000304 | -0.610941 |
| 66.250000 | -0.051083 | -1.089462 | 0.044412 | 0.065883 | 0.000599 | -0.011096 | -0.000316 | -0.634439 |
| 67.500000 | -0.052975 | -1.129813 | 0.045250 | 0.067516 | -0.000330 | -0.012908 | -0.000328 | -0.657937 |
| 68.750000 | -0.054867 | -1.170164 | 0.046087 | 0.069149 | -0.001293 | -0.014786 | -0.000340 | -0.681435 |
| 70.000000 | -0.056759 | -1.210514 | 0.046925 | 0.070782 | -0.002291 | -0.016730 | -0.000351 | -0.704933 |
| 71.250000 | -0.054868 | -1.170165 | 0.047763 | 0.070825 | -0.003288 | -0.016781 | -0.000340 | -0.681436 |
| 72.500000 | -0.052976 | -1.129814 | 0.048601 | 0.070868 | -0.004252 | -0.016830 | -0.000328 | -0.657938 |
| 73.750000 | -0.051084 | -1.089464 | 0.049439 | 0.070910 | -0.005182 | -0.016878 | -0.000316 | -0.634440 |
| 75.000000 | -0.049192 | -1.049113 | 0.050277 | 0.070953 | -0.006078 | -0.016923 | -0.000304 | -0.610942 |
| 76.750000 | -0.046354 | -0.988587 | 0.051450 | 0.070934 | -0.007273 | -0.016903 | -0.000287 | -0.575696 |
| 78.500000 | -0.043516 | -0.928061 | 0.052624 | 0.070914 | -0.008397 | -0.016884 | -0.000269 | -0.540449 |
| 80.250000 | -0.040678 | -0.867536 | 0.053797 | 0.070894 | -0.009450 | -0.016867 | -0.000252 | -0.505202 |
| 82.000000 | -0.037840 | -0.807010 | 0.054970 | 0.070874 | -0.010433 | -0.016850 | -0.000234 | -0.469955 |
| 84.500000 | -0.028380 | -0.605258 | 0.056646 | 0.068574 | -0.011616 | -0.015226 | -0.000175 | -0.352467 |
| 87.000000 | -0.018920 | -0.403506 | 0.058322 | 0.066274 | -0.012461 | -0.014066 | -0.000117 | -0.234978 |
| 89.500000 | -0.009460 | -0.201754 | 0.059998 | 0.063974 | -0.012969 | -0.013370 | -0.000058 | -0.117489 |
| 92.000000 | -0.000000 | -0.000000 | 0.061674 | 0.061674 | -0.013138 | -0.013138 | -0.000000 | -0.000000 |
| 94.000000 | 0.009459 | 0.201750 | 0.063014 | 0.059038 | -0.013002 | -0.013404 | 0.000058 | 0.117488 |
| 96.000000 | 0.018919 | 0.403503 | 0.064355 | 0.056403 | -0.012597 | -0.014201 | 0.000117 | 0.234976 |
| 98.000000 | 0.028379 | 0.605255 | 0.065696 | 0.053767 | -0.011920 | -0.015530 | 0.000175 | 0.352465 |
| 100.000000 | 0.037839 | 0.807007 | 0.067037 | 0.051132 | -0.010974 | -0.017391 | 0.000234 | 0.469954 |
| 103.000000 | 0.028569 | 0.609294 | 0.069048 | 0.057040 | -0.009550 | -0.013208 | 0.000177 | 0.354817 |
| 106.000000 | 0.019298 | 0.411576 | 0.071059 | 0.062947 | -0.008523 | -0.010192 | 0.000119 | 0.239678 |
| 109.000000 | 0.010027 | 0.213859 | 0.073070 | 0.068855 | -0.007894 | -0.008345 | 0.000062 | 0.124539 |
| 112.000000 | 0.000756 | 0.016141 | 0.075081 | 0.074763 | -0.007663 | -0.007665 | 0.000004 | 0.009400 |

D

RAY,AREA AND PRESSURE CALCULATIONS

FI= 20.000 NU= 69.415 COO= 1371.536 CO= 1371.536

| X | Y | Z | T | AREA | AGE | COS THETA |
|---|---|---|---|---|---|---|
| 0.49395E 03 | 0.18551E 03 | 0.28000E 05 | 0.10007E 04 | 0.64414E 03 | 0.32789E-02 | 0.72664E 00 |
| 0.24924E 04 | 0.93606E 03 | 0.26000E 05 | 0.10036E 04 | 0.33672E 04 | 0.71659E-02 | 0.73286E 00 |
| 0.45277E 04 | 0.17004E 04 | 0.24000E 05 | 0.10065E 04 | 0.63376E 04 | 0.94262E-02 | 0.73903E 00 |
| 0.66006E 04 | 0.24789E 04 | 0.22000E 05 | 0.10095E 04 | 0.95733E 04 | 0.11106E-01 | 0.74515E 00 |
| 0.87121E 04 | 0.32719E 04 | 0.20000E 05 | 0.10124E 04 | 0.13094E 05 | 0.12455E-01 | 0.75122E 00 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 0.10862E 05 | 0.40796E 04 | 0.18000E 05 | 0.10153E 04 | 0.16920E 05 | 0.13582E-01 | 0.75725E 00 |
| 0.13054E 05 | 0.49025E 04 | 0.16000E 05 | 0.10183E 04 | 0.21076E 05 | 0.14545E-01 | 0.76322E 00 |
| 0.15286E 05 | 0.57410E 04 | 0.14000E 05 | 0.10213E 04 | 0.25587E 05 | 0.15383E-01 | 0.76916E 00 |
| 0.17561E 05 | 0.65953E 04 | 0.12000E 05 | 0.10242E 04 | 0.30478E 05 | 0.16121E-01 | 0.77504E 00 |
| 0.19879E 05 | 0.74660E 04 | 0.10000E 05 | 0.10272E 04 | 0.35780E 05 | 0.16776E-01 | 0.78089E 00 |
| 0.22242E 05 | 0.83535E 04 | 0.80000E 04 | 0.10302E 04 | 0.41526E 05 | 0.17364E-01 | 0.78669E 00 |
| 0.24652E 05 | 0.92583E 04 | 0.60000E 04 | 0.10332E 04 | 0.47750E 05 | 0.17893E-01 | 0.79245E 00 |
| 0.27108E 05 | 0.10180E 05 | 0.40000E 04 | 0.10362E 04 | 0.54491E 05 | 0.18373E-01 | 0.79817E 00 |
| 0.28355E 05 | 0.10649E 05 | 0.30000E 04 | 0.10378E 04 | 0.58069E 05 | 0.18596E-01 | 0.80102E 00 |
| 0.29613E 05 | 0.11121E 05 | 0.20000E 04 | 0.10393E 04 | 0.61792E 05 | 0.18810E-01 | 0.80385E 00 |
| 0.32169F 05 | 0.12081E 05 | 0.00000E 00 | 0.10423E 04 | 0.69700E 05 | 0.19209E-01 | 0.80949E 00 |

| L | F | VE(L) | XI 0 | XI 1 | S INT | S | PI | DEL P |
|---|---|---|---|---|---|---|---|---|
| 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 1.125000 | 0.010500 | 0.238941 | 0.000754 | -0.003835 | 0.000090 | -0.000458 | 0.000062 | 0.125119 |
| 2.250000 | 0.021001 | 0.477883 | 0.001508 | -0.007671 | 0.000360 | -0.001833 | 0.000124 | 0.250239 |
| 3.374999 | 0.031501 | 0.716824 | 0.002262 | -0.011507 | 0.000810 | -0.004124 | 0.000187 | 0.375359 |
| 4.499999 | 0.042002 | 0.955765 | 0.003016 | -0.015342 | 0.001441 | -0.007332 | 0.000249 | 0.500479 |
| 5.874999 | 0.046637 | 1.061245 | 0.003938 | -0.016447 | 0.002371 | -0.008445 | 0.000277 | 0.555712 |
| 7.249999 | 0.051273 | 1.166724 | 0.004860 | -0.017551 | 0.003398 | -0.009676 | 0.000304 | 0.610945 |
| 8.625000 | 0.055908 | 1.272202 | 0.005781 | -0.018656 | 0.004522 | -0.011023 | 0.000332 | 0.666178 |
| 10.000000 | 0.060543 | 1.377681 | 0.006703 | -0.019760 | 0.005743 | -0.012486 | 0.000359 | 0.721412 |
| 10.875000 | 0.059597 | 1.356155 | 0.007290 | -0.018760 | 0.006545 | -0.011119 | 0.000354 | 0.710140 |
| 11.750000 | 0.058651 | 1.334629 | 0.007876 | -0.017760 | 0.007334 | -0.009773 | 0.000348 | 0.698868 |
| 12.625000 | 0.057705 | 1.313103 | 0.008463 | -0.016760 | 0.008110 | -0.008449 | 0.000343 | 0.687595 |
| 13.500000 | 0.056759 | 1.291576 | 0.009050 | -0.015760 | 0.008874 | -0.007147 | 0.000337 | 0.676323 |
| 15.124998 | 0.054111 | 1.231303 | 0.010139 | -0.013513 | 0.010248 | -0.004312 | 0.000321 | 0.644762 |
| 16.749996 | 0.051462 | 1.171030 | 0.011228 | -0.011265 | 0.011557 | -0.001613 | 0.000305 | 0.613200 |
| 18.374992 | 0.048813 | 1.110756 | 0.012318 | -0.009018 | 0.012800 | 0.000950 | 0.000290 | 0.581638 |
| 19.999988 | 0.046164 | 1.050483 | 0.013407 | -0.006771 | 0.013977 | 0.003378 | 0.000274 | 0.550077 |
| 22.499988 | 0.038880 | 0.884730 | 0.015083 | -0.001911 | 0.015599 | 0.008081 | 0.000231 | 0.463282 |
| 24.999988 | 0.031596 | 0.718978 | 0.016759 | 0.002948 | 0.016942 | 0.011977 | 0.000187 | 0.376487 |
| 27.499988 | 0.024312 | 0.553226 | 0.018435 | 0.007808 | 0.018008 | 0.015069 | 0.000144 | 0.289692 |
| 29.999988 | 0.017028 | 0.387473 | 0.020111 | 0.012668 | 0.018797 | 0.017355 | 0.000101 | 0.202897 |
| 30.999988 | 0.015609 | 0.355184 | 0.020781 | 0.013958 | 0.019046 | 0.017834 | 0.000092 | 0.185989 |
| 31.999988 | 0.014190 | 0.322894 | 0.021451 | 0.015249 | 0.019273 | 0.018272 | 0.000084 | 0.169081 |
| 32.999992 | 0.012771 | 0.290605 | 0.022122 | 0.016539 | 0.019479 | 0.018667 | 0.000075 | 0.152173 |
| 33.999992 | 0.011352 | 0.258315 | 0.022792 | 0.017830 | 0.019663 | 0.019022 | 0.000067 | 0.135265 |
| 35.499992 | 0.018919 | 0.430523 | 0.023798 | 0.015528 | 0.020009 | 0.018229 | 0.000112 | 0.225440 |
| 36.999992 | 0.026487 | 0.602734 | 0.024803 | 0.013225 | 0.020528 | 0.017039 | 0.000157 | 0.315616 |
| 38.499992 | 0.034055 | 0.774943 | 0.025809 | 0.010923 | 0.021221 | 0.015453 | 0.000202 | 0.405793 |
| 39.999992 | 0.041623 | 0.947154 | 0.026814 | 0.008620 | 0.022087 | 0.013471 | 0.000247 | 0.495969 |
| 40.999992 | 0.031218 | 0.710370 | 0.027485 | 0.013839 | 0.022642 | 0.017796 | 0.000185 | 0.371979 |
| 41.999992 | 0.020812 | 0.473581 | 0.028155 | 0.019058 | 0.023039 | 0.020885 | 0.000123 | 0.247987 |
| 42.999992 | 0.010406 | 0.236793 | 0.028825 | 0.024277 | 0.023277 | 0.022739 | 0.000061 | 0.123994 |
| 43.999992 | 0.000000 | 0.000004 | 0.029496 | 0.029496 | 0.023357 | 0.023357 | 0.000000 | 0.000002 |
| 45.499992 | -0.032163 | -0.731884 | 0.030501 | 0.044560 | 0.022989 | 0.017844 | -0.000191 | -0.383245 |
| 46.999992 | -0.064327 | -1.463776 | 0.031507 | 0.059625 | 0.021885 | 0.001306 | -0.000382 | -0.766494 |
| 48.499992 | -0.096491 | -2.195672 | 0.032512 | 0.074690 | 0.020045 | -0.026258 | -0.000573 | -1.149746 |
| 49.999992 | -0.128655 | -2.927564 | 0.033518 | 0.089754 | 0.017469 | -0.064848 | -0.000764 | -1.532995 |
| 51.499992 | -0.115412 | -2.626210 | 0.034524 | 0.084971 | 0.014677 | -0.051565 | -0.000686 | -1.375194 |
| 52.999992 | -0.102168 | -2.324842 | 0.035529 | 0.080188 | 0.012187 | -0.039724 | -0.000607 | -1.217385 |
| 54.499992 | -0.088924 | -2.023472 | 0.036535 | 0.075404 | 0.010001 | -0.029323 | -0.000528 | -1.059575 |
| 55.999992 | -0.075680 | -1.722104 | 0.037540 | 0.070621 | 0.008118 | -0.020365 | -0.000449 | -0.901766 |
| 56.999992 | -0.065274 | -1.485316 | 0.038211 | 0.066742 | 0.007043 | -0.014145 | -0.000388 | -0.777773 |
| 57.999992 | -0.054868 | -1.248527 | 0.038881 | 0.062864 | 0.006127 | -0.008844 | -0.000326 | -0.653781 |
| 58.999992 | -0.044462 | -1.011738 | 0.039551 | 0.058986 | 0.005369 | -0.004461 | -0.000264 | -0.529788 |
| 59.999992 | -0.034056 | -0.774948 | 0.040222 | 0.055108 | 0.004770 | -0.000997 | -0.000202 | -0.405795 |
| 61.249992 | -0.037839 | -0.861050 | 0.041060 | 0.057600 | 0.004085 | -0.003035 | -0.000224 | -0.450882 |
| 62.499992 | -0.041623 | -0.947155 | 0.041898 | 0.060092 | 0.003327 | -0.005288 | -0.000247 | -0.495970 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 63.749992 | -0.045407 | -1.033260 | 0.042736 | 0.062584 | 0.002497 | -0.007756 | -0.000269 | -0.541058 |
| 65.000000 | -0.049191 | -1.119364 | 0.043574 | 0.065076 | 0.001595 | -0.010438 | -0.000292 | -0.586140 |
| 66.250000 | -0.051083 | -1.162418 | 0.044412 | 0.066741 | 0.000639 | -0.012338 | -0.000303 | -0.608691 |
| 67.500000 | -0.052975 | -1.205471 | 0.045250 | 0.068406 | -0.000352 | -0.014309 | -0.000314 | -0.631235 |
| 68.750000 | -0.054867 | -1.248523 | 0.046087 | 0.070071 | -0.001380 | -0.016352 | -0.000326 | -0.653779 |
| 70.000000 | -0.056759 | -1.291576 | 0.046925 | 0.071736 | -0.002444 | -0.018466 | -0.000337 | -0.676323 |
| 71.250000 | -0.054868 | -1.248524 | 0.047763 | 0.071747 | -0.003508 | -0.018480 | -0.000326 | -0.653780 |
| 72.500000 | -0.052976 | -1.205472 | 0.048601 | 0.071758 | -0.004537 | -0.018494 | -0.000314 | -0.631235 |
| 73.750000 | -0.051084 | -1.162419 | 0.049439 | 0.071769 | -0.005529 | -0.018507 | -0.000303 | -0.608691 |
| 75.000000 | -0.049192 | -1.119367 | 0.050277 | 0.071780 | -0.006485 | -0.018519 | -0.000292 | -0.586147 |
| 76.750000 | -0.046354 | -1.054788 | 0.051450 | 0.071712 | -0.007760 | -0.018446 | -0.000275 | -0.552331 |
| 78.500000 | -0.043516 | -0.990209 | 0.052624 | 0.071645 | -0.008960 | -0.018377 | -0.000258 | -0.518515 |
| 80.250000 | -0.040678 | -0.925630 | 0.053797 | 0.071577 | -0.010083 | -0.018313 | -0.000241 | -0.484699 |
| 82.000000 | -0.037840 | -0.861051 | 0.054970 | 0.071510 | -0.011131 | -0.018252 | -0.000224 | -0.450882 |
| 84.500000 | -0.028380 | -0.645789 | 0.056646 | 0.069051 | -0.012394 | -0.016400 | -0.000168 | -0.338162 |
| 87.000000 | -0.018920 | -0.430527 | 0.058322 | 0.066592 | -0.013296 | -0.015076 | -0.000112 | -0.225442 |
| 89.500000 | -0.009460 | -0.215264 | 0.059998 | 0.064133 | -0.013837 | -0.014282 | -0.000056 | -0.112721 |
| 92.000000 | -0.000000 | -0.000001 | 0.061674 | 0.061674 | -0.014018 | -0.014018 | -0.000000 | -0.000000 |
| 94.000000 | 0.009459 | 0.215261 | 0.063014 | 0.058879 | -0.013873 | -0.014318 | 0.000056 | 0.112719 |
| 96.000000 | 0.018919 | 0.430523 | 0.064355 | 0.056085 | -0.013440 | -0.015221 | 0.000112 | 0.225440 |
| 98.000000 | 0.028379 | 0.645786 | 0.065696 | 0.053291 | -0.012719 | -0.016724 | 0.000168 | 0.338160 |
| 100.000000 | 0.037839 | 0.861048 | 0.067037 | 0.050496 | -0.011709 | -0.018830 | 0.000224 | 0.450881 |
| 103.000000 | 0.028569 | 0.650095 | 0.069048 | 0.056560 | -0.010189 | -0.014248 | 0.000169 | 0.340417 |
| 106.000000 | 0.019298 | 0.439137 | 0.071059 | 0.062623 | -0.009094 | -0.010946 | 0.000114 | 0.229951 |
| 109.000000 | 0.010027 | 0.228180 | 0.073070 | 0.068687 | -0.008423 | -0.008923 | 0.000059 | 0.119484 |
| 112.000000 | 0.000756 | 0.017222 | 0.075081 | 0.074750 | -0.008176 | -0.008179 | 0.000004 | 0.009018 |

GROUND INTERSECTION CURVES

| TIME | X | Y | PRESSURE |
|---|---|---|---|
| 1038.41284 | 29069.14848 | 8186.71485 | 1.65000 |
| 1038.41284 | 30318.26177 | 4814.55958 | 1.67215 |
| 1038.41284 | 30211.08207 | 1572.21753 | 1.65343 |

| TIME | X | Y | PRESSURE |
|---|---|---|---|
| 1039.27881 | 29164.34770 | 11564.28127 | 1.62998 |
| 1039.27881 | 30435.51957 | 8210.73830 | 1.65209 |
| 1039.27881 | 31675.45317 | 4781.64644 | 1.67431 |
| 1039.27881 | 31536.22270 | 1493.22436 | 1.65540 |
| 1039.27881 | 31458.94927 | -1726.83105 | 1.63686 |

| TIME | X | Y | PRESSURE |
|---|---|---|---|
| 1040.12671 | 29257.56645 | 14871.38283 | 1.61037 |
| 1040.12671 | 30550.33599 | 11536.06252 | 1.63245 |
| 1040.12671 | 31812.42192 | 8127.41700 | 1.65463 |
| 1040.12671 | 33013.53916 | 4736.84571 | 1.67689 |
| 1040.12671 | 32839.71104 | 1504.74341 | 1.65773 |
| 1040.12671 | 32728.77739 | -1758.81836 | 1.63901 |
| 1040.12671 | 32680.77739 | -4957.05274 | 1.62064 |

PROBLEM COMPLETED

E





NASA-Langley, 1969 — 2 CR-1299